

# Long-Run Risks, the Macroeconomy, and Asset Prices

By RAVI BANSAL, DANA KIKU AND AMIR YARON\*

Ravi Bansal and Amir Yaron (2004) developed the Long-Run Risk (LRR) model which emphasizes the role of long-run risks, that is, low-frequency movements in consumption growth rates and volatility, in accounting for a wide range of asset pricing puzzles. In this article we present a generalized LRR model, which allows us to study the role of cyclical fluctuations and macroeconomic crises on asset prices and expected returns. The Bansal and Yaron (2004) LRR model contains (i) a persistent expected consumption growth component, (ii) long-run variation in consumption volatility, and (iii) preference for early resolution of uncertainty. To evaluate the role of cyclical risks, we incorporate a cyclical component in consumption growth – this component is stationary in levels. To study financial market crises, we also entertain jumps in consumption growth and volatility.

We find that the magnitude of risk compensation for cyclical risks in consumption critically depends on the magnitude of the intertemporal elasticity of substitution (IES). When the IES is larger than one, cyclical risks carry a very small risk premium. When IES is close to zero, the risk compensation for cyclical variations is large; however, in this case the risk-free rate is implausibly high (in excess of 10 percent). It is, therefore, unlikely that the compensation for cyclical risks is of economically significant magnitude. This implication is also consistent with Robert E. Lucas (1987), who argues that economic costs of transient shocks are small and those for trend shocks are large.

To study financial market crises, we first explore jumps in the cyclical component of the generalized LRR model; Robert Barro, Emi Nakamura, Jón Steinsson and José Ursúa (2009) use a related LRR model for their analysis of crises. We find that even dramatic drops in growth of 10 percent, once every 5

years, have little impact on the risk premium. A more plausible way to model macroeconomic crises, in our view, is via small discrete reductions in long-run expected growth and/or a rise in aggregate uncertainty. These small discrete macroeconomic changes (i.e., jumps) in the generalized LRR model lead to large asset price movements and financial crises. Importantly, the model does not rely on implausible (over 20 percent) drops in consumption to trigger financial market crises.

Consistent with the LRR model, Lars Hansen, John Heaton and Nan Li (2008) and Ravi Bansal, Dana Kiku and Amir Yaron (2009) document predictable variation in consumption growth data at long horizons. Georg Kaltenbrunner and Lars Lochstoer (Forthcoming) show that a standard production-based model endogenously generates long-run predictable fluctuations in consumption growth.

Earlier work shows that the LRR model can explain an important set of asset pricing puzzles – for the term structure and related puzzles see Monika Piazzesi and Martin Schneider (2007), for credit spreads see Hui Chen (Forthcoming), for option prices see Itamar Drechsler and Amir Yaron (2007) and Bjørn Eraker and Ivan Shaliastovich (2008), and for cross-sectional differences in expected returns see Ravi Bansal, Robert F. Dittmar and Christian Lundblad (2005), Dana Kiku (2006), Hansen, Heaton and Li (2008) and Ravi Bansal, Robert F. Dittmar and Dana Kiku (2009). Ravi Bansal, Ronald Gallant and George Tauchen (2007) estimate the LRR model, and find considerable support for it.

## I. Generalized Long-Run Risks Model

We generalize the Bansal and Yaron (2004) model and allow for cyclical variations in aggregate consumption and dividends. Specifically, the level of log consumption ( $c_t$ ) consists of a deterministic trend, a stochastic trend ( $y_t$ ), and a transitory component ( $s_t$ ). That is,  $c_t = \mu t + y_t + s_t$ . The growth rate of the stochastic trend is assumed to follow:

$$(1) \quad \Delta y_{t+1} = x_t + \varphi_\eta \sigma_t \eta_{t+1} + J_{\eta,t+1},$$

\* Bansal: Fuqua School of Business, Duke University, 1 Towerview Drive, NC 27708, and National Bureau of Economic Research, ravi.bansal@duke.edu. Kiku: The Wharton School, University of Pennsylvania, 3620 Locust walk, PA 19104, kiku@wharton.upenn.edu. Yaron: The Wharton School, University of Pennsylvania, 3620 Locust walk, PA 19104, and National Bureau of Economic Research, yaron@wharton.upenn.edu.

where  $\eta_t$  is iid  $N(0, 1)$ , and  $J_{\eta,t}$  is a non-gaussian innovation. Thus, the evolution of consumption growth is given by:

$$(2) \quad \Delta c_{t+1} = \mu + x_t + \Delta s_{t+1} + \varphi_\eta \sigma_t \eta_{t+1} + J_{\eta,t+1}.$$

The dynamics of the state vector, denoted by  $\mathbf{Y}'_t = (x_t, s_t, \sigma_t^2 - \sigma_0^2)$ , are described by:

$$(3) \quad \mathbf{Y}_{t+1} = \Phi \mathbf{Y}_t + \mathbf{G}_t \mathbf{Z}_{t+1} + \mathbf{J}_{t+1},$$

where  $\text{diag}(\Phi) = (\rho_x, \rho_s, \nu)$ ,  $\text{diag}(\mathbf{G}_t) = (\varphi_e \sigma_t, \varphi_u \sigma_t, \sigma_w)$ , and the off-diagonal elements of the two matrices are zeros.  $\mathbf{Z}'_{t+1} = (e_{t+1}, u_{t+1}, w_{t+1})$  is a vector of iid standard gaussian shocks, and  $\mathbf{J}_{t+1}$  is a vector of jumps. The jump component of  $j$ -variable,  $j = \{\eta, x, s, \sigma^2\}$ , is modeled as a compound poisson process with a constant intensity and mean jump size of  $\bar{\mu}_j$ . Dividends are assumed to have levered exposure to consumption components, i.e., the log level of dividends follows:  $d_t = \mu_d t + \phi_y y_t + \phi_s s_t + \zeta_t$ , where  $\Delta \zeta_{t+1} = \varphi_d \sigma_t \varepsilon_{t+1}$ , and  $\varepsilon_t$  is an idiosyncratic dividend innovation drawn from  $N(0, 1)$ .

While Bansal and Yaron (2004) focus on variation in the persistent growth component  $x_t$ , we explore the asset pricing implications of cyclical variations in consumption driven by  $s_t$ . Following the LRR terminology, we refer to  $\eta_t$ ,  $x_t$  and  $\sigma_t$  as to short-run, long-run and volatility risks respectively; innovations in  $s_t$  are labeled cyclical risks. For the expositional simplicity, for the rest of this section, we assume that the jump component is absent.

The representative agent has Larry G. Epstein and Stanley E. Zin (1989) type recursive preferences. The log of the intertemporal marginal rate of substitution (IMRS), is given by:

$$(4) \quad m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1},$$

where  $r_{c,t+1}$  is the continuous return on the consumption asset,  $0 < \delta < 1$  reflects the agent's time preference,  $\gamma$  is the coefficient of risk aversion,  $\psi$  is the intertemporal elasticity of substitution (IES) and  $\theta = \frac{1-\gamma}{1-\frac{\gamma}{\psi}}$ . To derive the dynamics of asset prices we rely on log-linear analytical solutions. Specifically, we conjecture that the log of the price to consumption ratio follows,

$$(5) \quad z_t = A_0 + \mathbf{A}' \mathbf{Y}_t,$$

and solve for  $\mathbf{A}' = (A_x, A_s, A_{\sigma^2})$  using the Euler equation for the return on wealth. The loadings of the price-consumption ratio on the three state variables are given by:

$$(6) \quad \mathbf{A}' = \left( \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho_x}, \frac{(1 - \frac{1}{\psi})(\rho_s - 1)}{1 - \kappa_1 \rho_s}, \frac{0.5\theta H}{1 - \kappa_1 \nu} \right)$$

where  $\kappa_1$  is the constant of log-linearization of the wealth return, and  $H > 0$ . Bansal and Yaron (2004) show that as long as IES is larger than one, asset valuations rise with higher long-run expected growth  $x$ , and fall in response to an increase in consumption volatility. Moreover, when IES is greater than one, the elasticity of the price-consumption ratio with respect to the cyclical component is negative ( $A_s < 0$ ), capturing the standard mean-reversion intuition of business-cycle shocks. Note the difference between long-run and cyclical effects – when  $s$  is high, consumption is expected to fall due to anticipated mean-reversion to its trend (i.e., expected consumption growth is negative), whereas a positive innovation in  $x$  signals high consumption growth in the future. Thus, equity prices will react very differently to news about the long-run and cyclical consumption components. Similarly, real rates rise in response to positive  $x$ -shocks, however, they will drop in response to positive  $s$ -shocks. Therefore, in terms of real bonds,  $s$ -risk will contribute a positive risk premium, while  $x$ -risk will contribute a negative risk premium.

Given the solution for  $z_t$ , the dynamics of the log IMRS are described by:

$$(7) \quad m_{t+1} = m_0 + \mathbf{M}' \mathbf{Y}_t - \mathbf{A}' \zeta_{t+1}$$

where  $\zeta'_{t+1} = (\sigma_t \eta_{t+1}, \sigma_t e_{t+1}, \sigma_t u_{t+1}, \sigma_w w_{t+1})$ .  $\mathbf{A}$  represents the vector of the corresponding market prices of risks. Note that due to a separation between risk aversion and IES, each risk carries a separate premium. The expressions for  $\mathbf{M}$  and the market prices of risks, save for the cyclical component, are provided in Bansal and Yaron (2004). The IMRS loading on  $s_t$  is equal to  $(1 - \rho_s) \psi^{-1}$ , and the cyclical risk price  $\lambda_u$  equals  $[\gamma + (1 - \theta) \kappa_1 A_s] \varphi_u$ . Since  $\kappa_1 \approx 1$ ,  $\lambda_u$  is approximately equal to  $\psi^{-1} \varphi_u$ , that is, the market price of cyclical risks is decreasing in IES. Further, with a preference for early resolution of uncertainty (i.e.,  $\gamma > \psi^{-1}$ ), the price of long-run risks is positive. Finally, when jumps are incorporated, they also directly influence the IMRS and receive separate premia.

## II. Long-Run versus Business Cycle Risks

Table 1 presents the calibration output of the model that incorporates cyclical fluctuations in consumption without any jumps. We simulate the model on a monthly frequency and evaluate its implications using time-averaged annual data. The configuration of model parameters for consumption and dividends, stated below the table, is chosen to match annual data for the 1929-2008 period. Our calibration of the benchmark LRR model parameters is fairly close to the one in Bansal, Kiku and Yaron (2009). In the model specification with cycle, the persistence of the cyclical component is set at 0.96 and the magnitude of  $s$ -shocks is half the magnitude of iid-innovations to consumption growth.

TABLE 1—ASSET PRICING IMPLICATIONS OF THE LRR MODEL WITH CYCLICAL COMPONENTS

	LRR $\psi = 1.5$	LRR-Cycle $\psi = 1.5$	LRR-Cycle $\psi = 0.2$
Risk Premia	6.23	6.24	0.25
short-run risk	1.39	1.39	1.39
long-run risk	2.05	2.05	-1.03
volatility risk	2.79	2.79	-0.39
cycle risk	–	0.01	0.28
Return volatility	20.2	21.2	22.0
Risk-free Rate			
mean	1.25	1.24	10.5
volatility	1.04	1.14	8.13
Cons. Growth			
volatility	2.39	2.66	2.67
AC(1)	0.44	0.39	0.38

Notes: The model is calibrated using the following configuration:  $\delta = 0.999$ ,  $\gamma = 10$ ,  $\mu = \mu_d = 1.5e-3$ ,  $\varphi_\eta = 1$ ,  $\rho_x = 0.978$ ,  $\varphi_e = 0.035$ ,  $\rho_s = 0.96$ ,  $\varphi_s = 0.5$ ,  $\sigma_0 = 6e-3$ ,  $\nu = 0.999$ ,  $\sigma_w = 2e-6$ ,  $\phi_y = 2.5$ ,  $\phi_s = 2.5$ ,  $\phi_d = 6.2$ .

The second column (labeled LRR) is the benchmark case without cyclical risks; the model closely matches the consumption growth data, equity return and risk-free dynamics. The data counterparts are reported below in Table 2. The risk premium decomposition makes clear that the long-run growth risk and consumption volatility risk contribute the most to the equity risk premia. In the next column we report the results from the augmented model that includes the cyclical component. As it can be seen, with an IES greater than one, the market price of cyclical risks

is essentially zero and so is their contribution to the overall risk premia. The last column in Table 1 provides the output when the model contains the same cyclical process but the IES is equal to 0.2. In this case, the risk price for cyclical risk rises, while the magnitude of the other risk prices decreases. Note that the risk premia contribution of volatility and long-run risk is negative, and therefore the overall risk premium is negligible. Importantly, with this low IES configuration, the mean and volatility of the risk-free rate are implausible – 11 percent and 8 percent, respectively.

It follows that small values of IES raise the compensation for cyclical risk but fail to account for the observed dynamics of the risk-free rate. In addition, when IES is less than one, asset valuations rise with lower expected growth and higher consumption volatility. This configuration implies a negative premium for long-run and volatility risks as reported in the last column of Table 1. Empirical evidence in Ravi Bansal, Varoujan Khatchatrian and Amir Yaron (2005) shows that in the data, price-dividend ratios sharply drop in response to an increase in consumption volatility, and that asset valuations anticipate higher earnings growth; this evidence is consistent with an IES larger than one. In all, the empirical evidence and the model implications point to an IES that is larger than one, and hence the compensation for business cycle risks is close to zero.

To evaluate the ability of the LRR model to track the observed log price-dividend ratio, we utilize the LRR calibration of Table 1. Figure 1 displays the realized and the model implied price-dividend ratio. We first extract the two state variables  $x_t$  and  $\sigma_t^2$  in the data by projecting annual consumption growth onto the lagged price-dividend ratio and the risk-free rate – see Ravi Bansal, Dana Kiku and Amir Yaron (2007) for details. In an entirely analogous fashion, we estimate the state variables from inside the model using time-averaged annual quantities from the model-based simulation. We then regress the model's price-dividend ratio onto the model's extracted annual state variables,  $x_t$  and  $\sigma_t$ . The line referenced "Model" in Figure 1 is the fitted log price-dividend ratio using the model based price-dividend projection evaluated at the data based extracted state variables. Figure 1 shows that the model price-dividend ratio tracks that of the data quite well, including the declines in 1930 and 2008. Consistent with the LRR model, movements in measured expected growth and consumption volatility indeed drive asset prices.

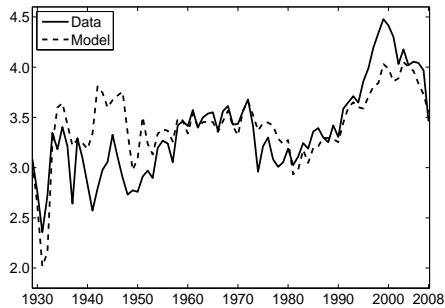


FIGURE 1. DATA AND MODEL PRICE-DIVIDEND RATIO

### III. Long-Run Risks and Crises

Table 2 provides a quantitative evaluation of the asset pricing implications of the two alternative views of macro-economic crises. In the first specification, an economic crisis is modeled as a negative jump in the cyclical component (as in Barro et al. (2009)). We refer to this specification as “Jumps in  $s$ ”. In the second case, macroeconomic crises are associated with a small but persistent reduction in the long-run consumption growth (jumps in  $x$ ) and a small but sustained rise in economic uncertainty (jumps in  $\sigma^2$ ). This specification is referred to as a model with “Jumps in  $x$  and  $\sigma^2$ ”. Apart from jumps, we rely on the same baseline calibrations with and without cyclical component as in Table 1 with IES=1.5. To facilitate the comparison between the two models, jump dynamics are chosen to yield a half-a-percent increase in the annual risk premia relative to those reported in Table 1.

As shown above, when IES is greater than one, any reasonably calibrated business cycle risks have a trivial effect on asset prices. Thus, generating a 50-basis-point increase in risk premia requires dramatically large declines in the cyclical component of consumption with a mean jump size of -12 percent on a monthly basis. Since historically the magnitude and frequency of such events are quite unlikely, this crisis specification fails to match the dynamics of observed consumption, significantly overshooting the volatility and higher moments of annual growth rates. Moreover, the price-dividend ratio will rise in response to a negative jump in the cyclical component (as  $A_s$  is negative)!

The last column of Table 2 reports key moments of consumption and asset prices implied by a model

TABLE 2—DYNAMICS OF GROWTH RATES AND PRICES

	Data	LRR Model with Jumps	
		in $s$	in $x$ & $\sigma^2$
Risk Premia	6.65	6.74	6.73
Risk-free Rate			
mean	0.90	1.00	1.21
volatility	1.85	2.53	1.02
Cons. Growth			
volatility	2.10	5.44	2.48
AC(1)	0.47	0.11	0.43

Notes: The model configuration is the same as in Table 1 with  $\psi = 1.5$ . In “Jumps in  $s$ ” column, the monthly parameter  $\bar{\mu}_s = -0.12$ . In the last column, the cycle component is shut down, jumps in  $x$  and  $\sigma^2$  occur simultaneously,  $\bar{\mu}_x = -2.5e-4$  and  $\bar{\mu}_{\sigma^2} = 3e-6$ . All jumps are drawn from the exponential distribution and, on average, arrive once in five years.

where crises are set off by small negative jumps in the long-run growth component and small positive jumps in the volatility of consumption growth. Since both risks carry sizable risk premia, this specification does not entail extreme fluctuations in growth rates and easily matches the dynamics of aggregate consumption. Note that although jumps, on average, are relatively small, they translate into large movements in asset prices. For example, a reduction in  $x$  that depresses consumption growth by half a percent per annum, and a 20 percent increase in annualized volatility will result in a 10 percent drop in the price-dividend ratio – comparable to the decline during 2008. Thus, empirically plausible macroeconomic events that lead to financial market crises are quite likely due to reductions in long-term expected growth and/or a rise in consumption volatility.

### IV. Conclusions

We present a generalized Long-Run Risks model, which incorporates a cyclical component and jumps. We argue that the compensation for cyclical risk is small. Significant cyclical risk premium requires low values of the intertemporal elasticity of substitution which are implausible as those lead to counterfactually high and volatile risk-free rates. We show that financial crises triggered by extreme declines in the cyclical component of consumption are empirically implausible. A more plausible view is that small but long-run declines in expected growth and/or an in-

crease in consumption volatility translate into financial crises. We show that the long-run risks model accounts for the dynamics of the observed price-dividend ratio quite well, including the crisis periods.

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