Intertemporal Capital Asset Pricing and the Fama-French Three-Factor Model*

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Abstract

Characterizing the instantaneous investment opportunity set by the real interest rate and the maximum Sharpe ratio, a simple model of time varying investment opportunities is posited in which these two variables follow correlated Ornstein-Uhlenbeck processes, and the implications for stock and bond valuation are developed. The model suggests that the prices of certain portfolios that are related to the Fama-French HML and SMB hedge portfolio returns will carry information about investment opportunities. This provides a justification for the risk premia that have been found to be associated with these hedge portfolio returns. Evidence that the FF portfolios are in fact associated with variation in the investment opportunity set is found from an analysis of stock returns. Further evidence of time variation in the real investment opportunity set is found by analyzing bond yields, and the time variation in investment opportunities that is identified from bond yields is shown to be associated both with the time-variation in investment opportunities that is identified from stock returns and with the returns on the Fama-French hedge portfolios. Finally, it is shown that the estimated parameters imply substantial variation in stock prices that is not associated with cash flow expectations.

1 Introduction

In the short run, investment opportunities depend only on the real interest rate and the slope of the capital market line, or Sharpe ratio, as in the classic Sharpe-Lintner Capital Asset Pricing Model. The slope of the capital market line depends in turn on the risk premium and volatility of the market return, and there is now strong evidence of time variation both in the equity risk premium and in market volatility, implying variation in the market Sharpe ratio, as well as in the real interest rate. Kandel and Stambaugh (1990), Whitelaw (1997), and Perez-Quiros and Timmermann (2000) all demonstrate significant cyclical variation in the market Sharpe ratio.[1] Given the evidence of time variation in short-run investment opportunities, four questions present themselves. First, how should future cash flows be valued when the investment opportunity set varies over time? Since Merton (1973) it has been clear that the empirically challenged single period CAPM is unlikely to provide reliable guidance under these circumstances although, as Cornell *et al.* (1997, p12) point out, the CAPM is the only asset pricing model that has been applied widely in practice. Secondly, is it possible that it is time variation in investment opportunities that accounts largely for the empirical failure of the single period CAPM as Merton's analysis would suggest,[2] and is it possible that the empirical success of the Fama-French three factor model[3] is due to the ability of this model to capture time variation in investment opportunities?[4] Thirdly, to what extent is variation in stock prices due to variation in investment opportunities rather than to variation in cash flow

[1]Other studies that identify significant predictors of the equity risk premium include: Lintner (1975), Fama and Schwert (1977) for interest rates; Campbell and Shiller (1988) and Fama and French (1988) for dividend yield; Fama and French (1989) for term spread and junk bond yield spread; Kothari and Shanken (1999) for Book-to-Market ratio.

[2]It is possible for the single period CAPM to hold even with time-varying investment opportunities. See Rubinstein (1976) and Constantinides (1980).

[3]Daniel and Titman (1997) question the role of the FF portfolios, but Davis *et al.* (2000) confirm the original FF findings using a larger data set.

[4]Lewellen (2000, p38) remarks that "the risk factors captured by the size and B/M mimicking portfolios have not been identified. The rational pricing story will remain incomplete, and perhaps unconvincing, until we know more about the underlying risks."

expectations?[5] Fourthly, can consideration of time variation in investment opportunities account for the finding that individual stock betas show little dependence on earnings or cash flow covariances with the market (Campbell and Mei, 1993), but are strongly related to the duration of equity cash flows (Cornell, 1999)?

In this paper, we are concerned with the first three issues: how cash flows should be valued when there is time-variation in investment opportunities, the relation between time variation in investment opportunities and the Fama-French three factor model, and the variation in stock prices due to variation in the investment opportunity set. First, we develop a parsimonious model of cash flow valuation that takes account of time-variation in investment opportunities and show how this can be used to value equity claims, and indexed as well as nominal bonds.

To address the second issue, we use the model to show that the (scaled) prices of the portfolios that Fama and French use to construct their HML and SMB hedge portfolio returns, as well as the term spread, are likely to contain information about investment opportunities. Using data on US stock returns, we find that these variables do indeed have predictive power for both the real interest rate and the Sharpe ratio. This finding is consistent with the existence of the risk premia that Fama and French have found to be associated with loadings on these hedge portfolio returns, if risk premia are determined by a Merton (1973) type Intertemporal Capital Asset Pricing Model (ICAPM). We also apply the model to the yields of pure discount Treasury bonds and find further evidence of time variation in the estimated market Sharpe ratio as well as in the real interest rate. Furthermore, we show that the estimates of the investment opportunity set statistics that are derived from the bond yield data are related, both to estimates of these statistics that are derived from the equity market data, and to the scaled prices of the Fama-French portfolios, and that innovations in the estimated opportunity set statistics are correlated with the returns on the Fama-French hedge portfolios, HML and SMB. We conclude that

[5] This issue has been examined previously by Campbell using a VAR framework in a series of papers including Campbell and Ammer (1993) and Campbell and Shiller (1988).

at least a part of the risk premia associated with these portfolios can be attributed to their role as investment opportunity set hedges.

Thirdly, using the estimated parameters of the stochastic process of the investment opportunity set, we show that a significant proportion of the variation in stock prices can be attributed to variation in the opportunity set state variables rather than to variation in cash flow expectations.

At least three alternative explanations have been offered previously for the empirical success of the Fama-French three-factor model. These are based respectively on: problems in the measurement of beta, the ICAPM, and the APT. Berk, Green and Naik (1999) and Gomes, Kogan, and Zhang (2000) develop models that explain the Fama-French results on the basis of problems in the measurement of beta. In these models firm betas[6] are stochastic, and there is a statistical relation between average returns, *unconditional* betas and other firm characteristics such as size and book-to-market ratio, which could be captured by a model such as the Fama-French three-factor model. Fama and French (FF) themselves have suggested the ICAPM as one possible reason for the premia that they find to be associated with loadings on the SMB and HML hedge portfolios that are formed on the basis of firm size and book-to-market ratio. In FF (1995) they argue that the premia, "are consistent with a multi-factor version of Merton's (1973) intertemporal asset pricing model in which size and BE/ME proxy for sensitivity to risk factors in returns." They have also suggested an APT interpretation, arguing that "if the size and BE/ME risk factors are the results of rational pricing, they must be driven by common factors in shocks to expected earnings that are related to size and BE/ME." We call this second argument the "APT story" since, in contrast to the ICAPM, it provides an essentially single period rationale for the premia associated with these portfolios. FF find little support for the APT story.[7]

[6]In these papers betas are measured with respect to the pricing kernel.

[7]However, in results not reported here, we also provide some supportive evidence for the APT story, by showing that the FF portfolio returns are associated with returns on assets that are not included in the

Other authors have also suggested that the Fama-French portfolios may be related to the investment opportunity set and that their risk premia may therefore be justified by appeal to the ICAPM. For example, Liew and Vassalou (2000) report that annual returns on the HMB and SML hedge portfolios predict GDP growth in several countries. Vassalou (1999) shows, however, that replacing the returns on the HML and SMB portfolios by their predicted innovation in GNP leads to a significant degradation of the asset pricing power of the Fama-French three factor model, implying that the asset pricing power of the three factor model does not rest on the ability of the two hedge portfolio returns to predict GNP growth. This is not very surprising even if there is an ICAPM explanation for the premia associated with these portfolios. First, priced state variables must predict variation in the financial investment opportunity set, and this variation is not necessarily related to changes in GNP growth. Secondly, returns on these portfolios cannot in themselves be state variables since their continuous time limit is a (geometric) Brownian motion.[8] Chen (2001) also attempts to test whether the rewards to covariance with returns on the HMB and SML portfolios (as well as a momentum factor) are consistent with rational asset pricing by examining whether the predictive power of the (discrete-time) returns on these portfolios for the returns on aggregate wealth is sufficient to warrant their risk premium. His paper also ignores the fact that returns on these portfolios cannot be state variables.

While previous authors have suggested that scaled asset prices may have predictive power for returns,[9] we suggest in this paper that *ratios* of scaled asset prices have predictive power. We also construct a simple valuation model with time-varying riskless rates and risk premia, and relate this to cross-sectional asset pricing results,[10] and relate bond market based estimates of the Sharpe ratio and real interest rate to the FF hedge portfolio

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conventional measure of the (stock) market portfolio. Consistent with this, Heaton and Lucas (2000) find evidence that the inclusion of entrepreneurial income in an asset pricing model reduces the importance of the FF portfolios. See also Polk (1998).

[8]However, it is possible that discrete time returns on the portfolios are noisy signals of the state variables that determine the drifts of the portfolio returns.

[9]See Ball (1978), Miller and Scholes (1982), Berk (1995), and Kothari and Shanken (1997).

[10]Campbell (1993) also points out that state variables that are priced in the cross-section must, in a rational model, have predictive power - in his case for future consumption.

returns.

The remainder of the paper is organized as follows. In Section 2, we construct a simple valuation model with a stochastic interest rate and a stochastic Sharpe ratio. In Section 3, we specialize the model to the ICAPM and show that ratios of security prices can be used as instruments for the state variables that determine the short term investment opportunity set. In Section 4, we describe the estimation approaches and the construction of the data that are used in the subsequent empirical tests. Empirical results are reported in Section 5. Section 6 provides some illustrative calibration results which suggest that time-varying discount factors are a major determinant of the risk of equity returns. Section 7 concludes.

2 Valuation with Stochastic Investment Opportunities

The value of a claim to a future cash flow depends on both the characteristics of the cash flow itself, its expected value, time to realization, and risk, and on the macroeconomic environment as represented by interest rates and risk premia. Holding the risk characteristics of the cash flow constant, unanticipated changes in claim value will be driven by changes in both the expected value of the cash flow and in interest rates and risk premia. Most extant valuation models place primary emphasis on the role of cash flow related risk. However, Campbell and Ammer (1993) estimate that only about 15% of the variance of aggregate stock returns is attributable to news about future dividends. Their results further show that news about real interest rates plays a relatively minor role, leaving about 70% of the total variance of stock returns to be explained by news about future excess returns or risk premia. Fama and French (1993)[11] demonstrate that there is considerable common variation between bond and stock returns, which is also consistent

[11]See also Cornell (1999).

with common variation in real interest rates and risk premia. In this section we construct an explicit model for the valuation of stochastic cash flows, taking account of stochastic variation in interest rates and risk premia.

Let V denote the value of a non-dividend paying asset. Then a pricing kernel is a random variable, m, such that $E[d(mV)] = 0$.[12] This implies that the expected return on the asset is given by:

$$\mathrm{E}\left[\frac{dV}{V}\right] = -\mathrm{E}\left[\frac{dm}{m}\right] - \mathrm{cov}\left(\frac{dm}{m}, \frac{dV}{V}\right) \tag{1}$$

Assume that the dynamics of the pricing kernel can be written as a diffusion process:

$$\frac{dm}{m} = -\mathrm{r}(X)dt - \eta(X)dz_m \tag{2}$$

where X is a vector of variables that follow a vector Markov diffusion process:

$$dX = \mu_X dt + \sigma_X dz_X \tag{3}$$

Then equations (1) and (2) imply that the expected return on the asset is given by:

$$\mathrm{E}\left[\frac{dV}{V}\right] \equiv \mu_V dt = r(X)dt + \eta(X)\rho_{Vm}\sigma_V dt \tag{4}$$

where $\rho_{Vm}dt = dz_V dz_m$, and σ_V is the volatility of the return on the asset. It follows that $r(X)$ is the riskless rate since it is the return on an asset with $\sigma_V = 0$, and $\eta(X)$ is the risk premium per unit of covariance with the pricing kernel. It is immediate from equation (4), that the Sharpe ratio for any asset V is given by $S_V \equiv (\mu_V - r)/\sigma_V = \eta\rho_{Vm}$. Recognizing

[12]See Cochrane (2001) for a complete treatment.

that ρ_{Vm} is a correlation coefficient, it follows that η is the maximum Sharpe ratio for any asset in the market - it is the "market" Sharpe ratio. An investor's *instantaneous* investment opportunities then are fully described by the vector $(r, \eta)'$, the instantaneously riskless rate and the Sharpe ratio of the capital market line.

In order to construct a tractable model we shall simplify by identifying the vector X with $(r, \eta)'$, and assume that r and η follow simple correlated Ornstein-Uhlenbeck processes.[13] Then, since the expected returns on all securities are functions only of $(r, \eta)'$, the dynamics of the investment opportunity set are fully captured by:

$$\frac{dm}{m} = -rdt - \eta dz_m \tag{5.1}$$

$$dr = \kappa_r(\overline{r} - r)dt + \sigma_r dz_r \tag{5.2}$$

$$d\eta = \kappa_\eta(\overline{\eta} - \eta)dt + \sigma_\eta dz_\eta \tag{5.3}$$

The structure (5) implies that the riskless interest rate is stochastic, and that all risk premia are proportional to the stochastic Sharpe ratio η. To analyze the asset pricing implications of the system (5), consider a claim to a (real) cash flow, x, which is due at time T. Let the expectation at time t of the cash flow be given by $Y(t) \equiv \mathrm{E}\left[x|\Lambda_t\right]$ where Λ_t is the information available at time t, and $Y(t)$ follows the driftless geometric Brownian motion with constant volatility, σ_Y:[14]

$$\frac{dY}{Y} = \sigma_Y dz_Y \tag{6}$$

[13] Kim and Omberg (1996) also assume an O-U process for the Sharpe ratio. For a structural model of time variation in investment opportunities that relies on habit formation see Campbell and Cochrane (1999).

[14] The assumption of constant volatility is for convenience only. For example, as Samuelson (1965) has shown, the volatility of the expectation of a future cash flow will decrease monotonically with the time to maturity if the cash flow has a mean-reverting component.

Letting ρ_{ij} denote the correlation between dz_i and dz_j, the value of the claim to the cash flow is given in the following theorem.

Theorem 1 *In an economy where the investment opportunity set is described by (5), the value at time t of a claim to a real cash flow x at time T, whose expectation, y, follows the stochastic process (6), is given by:*

$$V(y,\tau,r,\eta) = \mathrm{E}_t^Q \left[x_T \exp^{-\int_t^T r(s)ds} \right] = \mathrm{E}_t^Q \left[y_T \exp^{-\int_t^T r(s)ds} \right] = y_t v(\tau,r,\eta) \tag{7}$$

where Q denotes the risk neutral probability measure, and

$$v(\tau,r,\eta) = \exp[A(\tau) - B(\tau)r - D(\tau)\eta] \tag{8}$$

with

$$B(\tau) = \kappa_r{}^{-1}(1 - e^{-\kappa_r \tau}) \tag{9}$$

$$D(\tau) = d_1 + d_2 e^{-\kappa_\eta^* \tau} + d_3 e^{-\kappa_r \tau} \tag{10}$$

$$\begin{aligned} A(\tau) &= a_1\tau + a_2 \frac{1 - e^{-\kappa_r \tau}}{\kappa_r} + a_4 \frac{1 - e^{-\kappa_\eta^* \tau}}{\kappa_\eta^*} + a_5 \frac{1 - e^{-2\kappa_r \tau}}{2\kappa_r} \\ &+ a_7 \frac{1 - e^{-2\kappa_\eta^* \tau}}{2\kappa_\eta^*} + a_8 \frac{1 - e^{-(\kappa_\eta^* + \kappa_r)\tau}}{\kappa_\eta^* + \kappa_r}. \end{aligned} \tag{11}$$

where $\kappa_\eta^* \equiv \kappa_\eta + \sigma_\eta \rho_{m\eta}$, d_1, d_2, d_3 are given in equations (A16)-(A18) by setting $\rho_{mP} = \rho_{m\pi} = 0$, and $a_1, \dots, a_8$ are given in equations (A24)-(A31) by setting σ_P, σ_π, and $\bar{\pi}$ to zero.

Theorem 1 implies that the value per unit of expected payoff of the claim is a function of the maturity τ, and the covariance with the pricing kernel, or systematic risk, $\sigma_Y \rho_{Ym}$, of the underlying cash flow, as well as of the two stochastic factors that determine the investment opportunity set, r and η.

Lemma 1 *In the "positive cash flow beta" case in which $\rho_{Ym} > 0$,[15] $B(\tau)$ and $D(\tau)$ are positive and increase with τ, the time to maturity of the cash flow, provided that there is a positive risk premium for interest rate risk ($\rho_{mr} < 0$).[16]*

Lemma 1 characterizes the dependence of the factor sensitivities (measured by the semi-elasticities of claim value) on the cash flow maturity, τ. The longer the cash flow maturity, the more sensitive is the value of the claim to shocks in the interest rate and risk premium.

Following Theorem 1 and applying Ito's Lemma, the return on a claim can be written as:

$$\frac{dV}{V} = \mu(r, \eta, \tau)dt + \frac{dY}{Y} - B(\tau)\sigma_r dz_r - D(\tau)\sigma_\eta dz_\eta. \tag{12}$$

The expected return is shown in the Appendix to be given by:

$$\mu \equiv \mu(r, \eta, \tau) = r + (D_\tau(\tau) + \kappa_\eta D(\tau))\eta = r + h(\tau)\eta, \tag{13}$$

where $h(\tau)$ is the asset's risk premium expressed relative to the market Sharpe ratio. The form of the risk premium expression (13) can be understood by noting that,

[15] *This is the condition for the risk premium associated with innovations in Y to be positive.*

[16] *This condition for bonds that are more exposed to interest rate risk to have higher risk premia is satisfied by the empirical estimates reported in Table 5 below.*

under the assumptions we have made, the claim value can also be written as $V = \mathrm{E}\left[Y_T e^{-\int_t^T (r_s + h(T-s)\eta_s) ds}\right]$. Noting that $D = -\frac{V_\eta}{V}$, differentiation of this expression with respect to η_t implies $D = \int_t^T h(T-s) e^{-\kappa_\eta (s-t)} ds$, which then leads to the expression for $h(\tau)$ in equation (13).

Lemma 2 *(i) In the positive cash flow beta case ($\rho_{Ym} > 0$), if $\rho_{mr} < 0$ and $\rho_{m\eta} < 0$, then the expected return on the claim increases with the cash flow maturity;*

(ii)If, in an N-security market, the cash flow expectations, $Y_i, (i = 1, \ldots, N)$, have a K-factor structure, then the returns have an (K+2) factor structure, with the two additional factors corresponding to innovations in the opportunity set variables, r and η.

The restrictions imposed on the pricing kernel ($\rho_{mr} < 0$ and $\rho_{m\eta} < 0$) in Part (i) of the Lemma, which are satisfied by the empirical estimates reported in Table 5 below, imply that there are positive risk premia associated with exposure to real interest rate and Sharpe ratio risk.[17]

Lemma 3 *In the "zero cash flow beta" case in which $\rho_{Ym} = 0$, the value of the claim is given by $V(Y, \tau, r, \eta) \equiv Y v(\tau, r, \eta)$, and*

$$v(\tau, r, \eta) = \exp[A^*(\tau) - B(\tau)r - D^*(\tau)\eta] \quad (14)$$

where $A^*(\tau)$ and $D^*(\tau)$ are obtained by setting ρ_{Ym} equal to zero in expressions (11) and (10).

[17]We are assuming here that the claim values decrease with an increase in the real interest rate or Sharpe ratio.

A special case of Lemma 3 applies for a real discount bond for which $x \equiv Y \equiv 1$. Then expression (14) generalizes the Vasicek (1977) model for the price of a (real) unit discount bond of maturity τ to the case in which the risk premium as well as the interest rate, is stochastic. In order to value *nominal* bonds, it is necessary to specify the stochastic process for the price level, P. We assume that the price level follows the diffusion:

$$\frac{dP}{P} = \pi dt + \sigma_P dz_P, \tag{15}$$

where the volality of inflation, σ_P, is constant, while the expected rate of inflation, π, follows an Ornstein-Uhlenbeck process:

$$d\pi = \kappa_\pi(\overline{\pi} - \pi)dt + \sigma_\pi dz_\pi. \tag{16}$$

Then, noting that the real payoff of the nominal bond is $1/P_T$, the nominal price of a zero coupon bond with a face value of $\$1$ and maturity of τ, $N(P, r, \pi, \eta, \tau)$, and the corresponding real price, $n(P, r, \pi, \eta, \tau)$, are stated in the following theorem.

Theorem 2 *If the stochastic process for the price level P is as described by (15) and (16), the nominal and the real prices of a zero coupon bond with face value of $\$1$ and maturity τ, are:*

$$N(P, r, \pi, \eta, \tau) \equiv Pn(r, \pi, \eta, \tau) = \exp[\widehat{A}(\tau) - B(\tau)r - C(\tau)\pi - \widehat{D}(\tau)\eta] \tag{17}$$

where

$$B(\tau) = \kappa_r^{-1}\left(1 - e^{-\kappa_r \tau}\right) \tag{18}$$

$$C(\tau) = \kappa_\pi^{-1}\left(1 - e^{-\kappa_\pi \tau}\right) \tag{19}$$

$$\widehat{D}(\tau) = \hat{d}_1 + \hat{d}_2 e^{-\kappa_\eta^* \tau} + \hat{d}_3 e^{-\kappa_r \tau} + \hat{d}_4 e^{-\kappa_\pi \tau} \tag{20}$$

$$\begin{aligned} \widehat{A}(\tau) = {} & \hat{a}_1 \tau + \hat{a}_2 \frac{1 - e^{-\kappa_r \tau}}{\kappa_r} + \hat{a}_3 \frac{1 - e^{-\kappa_\pi \tau}}{\kappa_\pi} + \hat{a}_4 \frac{1 - e^{-\kappa_\eta^* \tau}}{\kappa_\eta^*} \\ & + \hat{a}_5 \frac{1 - e^{-2\kappa_r \tau}}{2\kappa_r} + \hat{a}_6 \frac{1 - e^{-2\kappa_\pi \tau}}{2\kappa_\pi} + \hat{a}_7 \frac{1 - e^{-2\kappa_\eta^* \tau}}{2\kappa_\eta^*} \\ & + \hat{a}_8 \frac{1 - e^{-(\kappa_\eta^* + \kappa_r)\tau}}{\kappa_\eta^* + \kappa_r} + \hat{a}_9 \frac{1 - e^{-(\kappa_\eta^* + \kappa_\pi)\tau}}{\kappa_\eta^* + \kappa_\pi} + \hat{a}_{10} \frac{1 - e^{-(\kappa_r + \kappa_\pi)\tau}}{\kappa_r + \kappa_\pi} \end{aligned} \tag{21}$$

$\kappa_\eta^* \equiv \kappa_\eta + \sigma_\eta \rho_{m\eta}$, *and* $\hat{d}_1, \ldots, \hat{d}_4$, $\hat{a}_1, \ldots, \hat{a}_{10}$ *are constants whose values are given in equations (A16)-(A19) and (A24)-(A33) by setting* $\sigma_Y = 0$.

Finally, the yield of the bond is given by:

$$-\frac{\ln N}{\tau} = -\frac{\widehat{A}(\tau)}{\tau} + \frac{B(\tau)}{\tau} r + \frac{C(\tau)}{\tau} \pi + \frac{\widehat{D}(\tau)}{\tau} \eta. \tag{22}$$

3 Intertemporal Asset Pricing and the Fama-French Portfolios

While the valuation model (5) explicitly allows for time-variation in the investment opportunity set, it is not equivalent to Merton's ICAPM without further specification of the covariance characteristics of the pricing kernel. For example, the valuation model will satisfy the simple CAPM if the innovation in the pricing kernel is perfectly correlated with the return on the market portfolio. A specific version of the ICAPM is obtained by specializing the pricing system (5) so that the innovation in the pricing kernel is an exact linear function of the market return and the innovations in r and η:

$$\frac{dm}{m} = -rdt - \omega\eta\zeta' dz \tag{23}$$

where $\zeta' = (\zeta_M, \zeta_\eta, \zeta_r)', dz = (dz_M, dz_\eta, dz_r)', \omega \equiv (\zeta'\Omega\zeta)^{-1/2}$, and $\Omega dt = (dz)(dz)'$, where M denotes the market portfolio.

Then, using the definition of the pricing kernel (1), and equation (23), the expected return on security i, μ_i may be written as:

$$\mu_i = r + \eta\omega\zeta'\sigma_i \tag{24}$$

where σ_i is the (3×1) vector of covariances of the security return with the market return and the innovations in the state variables, r and η. This is simply a restatement of the ICAPM.

Note that, while the state variables of the ICAPM described by equations (5) and (23), the Sharpe ratio, η, and the real interest rate, r, are not directly observable,[18] the pricing model expressed in equations (7) and (8) implies that the log of the ratio of the values of any two claims i and j can be expressed as the sum of the log ratio of the expected (real) cash flows, a time and risk-dependent constant, and linear functions of the investment opportunity set parameters r and η:

$$\ln\left(\frac{V_i}{V_j}\right) = \ln\left(\frac{Y_i}{Y_j}\right) + [A_i - A_j] - [B_i - B_j]\, r - [D_i - D_j]\, \eta \tag{25}$$

[18] r would be observable if short term indexed bonds were traded.

Moreover, equation (22) implies that the yield to maturity on a nominal bond of maturity τ, $R(\tau) \equiv -\ln N(\tau, r, \eta)/\tau$ is also a linear function of r, and η, as well as the expected rate of inflation, π. Thus, corresponding to equation (25), the yield spread between bonds with maturities τ_1 and τ_2 can be written as:

$$\begin{aligned} R(\tau_1) - R(\tau_2) &= \left[\frac{\widehat{A}(\tau_2)}{\tau_2} - \frac{\widehat{A}(\tau_1)}{\tau_1}\right] + \left[\frac{B(\tau_1)}{\tau_1} - \frac{B(\tau_2)}{\tau_2}\right] r \\ &\quad + \left[\frac{C(\tau_1)}{\tau_1} - \frac{C(\tau_2)}{\tau_2}\right] \pi + \left[\frac{\widehat{D}(\tau_1)}{\tau_1} - \frac{\widehat{D}(\tau_2)}{\tau_2}\right] \eta \end{aligned} \tag{26}$$

Since the (log) value ratios are functions of the state variables, (r, η), covariances with innovations in the value ratios will correspond to covariances with linear combinations of innovations in the state variables. Therefore, under the ICAPM, covariances with innovations in the price ratios should be priced.

Equation (25) provides a theoretical rationale for the empirical importance of the HML and SMB hedge portfolios in the FF three-factor model since it implies a relation between the returns on these portfolios and innovations in r and η. Thus, letting H and L denote portfolios of high and low B/M firms, and letting R_H denote the (discrete time) return on portfolio H etc., equation (25) implies the following approximate relation between the return on HML, R_{HML}, the changes in the values of the H and L portfolios, and innovations in r and η:

$$R_{HML} \approx \triangle \ln V_H - \triangle \ln V_L \approx -(B_H - B_L)\triangle r - (D_H - D_L)\triangle \eta + u \tag{27}$$

where $u \equiv \triangle \ln Y_H - \triangle \ln Y_L$ is the noise introduced by the difference between the changes in cash flow expectations for the two portfolios. Hence, if $B_H \neq B_L$ and $D_H \neq D_L$,[19] the

[19] Since the B/M ratio is associated with growth, or the duration of firm cash flows, we should expect from Lemma 1 that firms with different B/M ratios will have different sensitivities to r and η. Moreover, Perez-Quiros and Timmermann (2000) show that portfolios of large and small firms have different sensitivities to

covariance of a security return with R_{HML} will be a linear combination of its covariances with the state variable innovations $\triangle\eta$ and $\triangle r$, plus a term related to the noise component, u. Similarly, the covariance with R_{SMB} will provide a second noisy linear combination of covariances with the state variable innovations. Therefore, if the prices of portfolios of large and small firms and of high and low B/M firms show reliable predictive power for the real interest rate, r, and equity premium or Sharpe ratio, η, we should expect a cross-sectional relation between expected returns and factor loadings on the corresponding hedge portfolio returns as FF have found. In Section 5 we shall examine the predictive power of these portfolio price ratios.

4 Data and Estimation

We shall adopt two distinct approaches to determining whether the Fama-French hedge portfolio returns are related to innovations in the investment opportunity set, as they must be if the risk premia associated with them are to be explained by the ICAPM. First, we shall test whether portfolio price ratios that are related to the FF hedge portfolio returns (together with the term spread) predict real riskless returns and the Sharpe ratio: these tests, while motivated by the model developed in the previous section, do not rely on the specific functional forms derived there. Secondly, we shall apply the exact model of equation (22) to bond yields and extract the time series of the state variables, r, η, and π, using a Kalman filter, and then test whether these state variable estimates are related to the Fama-French hedge portfolio returns.

The first approach is based on equations (25) and (26) which suggest that linear combinations of pairs of log value ratios or term spreads can be regarded as (noisy) instruments of the (real) investment opportunity set state variables, r and η. In the case

credit conditions, so that we should expect them to have different loadings on r at least.

of the log value ratios, noise is introduced by the omission of the log expected cash flow ratios, $\ln(Y_i/Y_j)$, and in empirical applications approximation error will be introduced by the use of prices of assets that are claims to streams of cash flows rather than to single dated cash flows. In the case of empirical term spreads, noise will be introduced if the bonds are nominal rather than real, and approximation error will be introduced if coupon bonds are used in place of the theoretically required discount bond yields. Given these errors, it is natural to think of using several price ratios or yield spreads as proxies for the state vector (r, η). Therefore we estimate the investment opportunity set variables, r and η, by regressing the realized values of the real interest rate and (normalized) market excess return on the state variable instruments and calculating the fitted values from these regressions. Since the portfolio prices (values) are non-stationary, we shall scale the prices by the book values of the portfolios; this should also alleviate the problem caused by the non-observability of the expected cash flow ratio.

The second approach to estimating the dynamics of the investment opportunity set is to employ a Kalman filter applying to data on inflation and bond yields together with the theoretical relation, equation (22), to estimate the unobservable state variables, r, π and η, and their dynamics. Details of the estimation are presented in Appendix C. In summary, there are n observation equations based on the yields at time t, $y_{\tau_j,t}$, on bonds with maturities τ_j, $j = 1, \cdots, n$, which are the same as equation (22) except for the measurement error terms, ϵ_{τ_j}:

$$y_{\tau_j,t} \equiv -\frac{\ln N(t, t+\tau_j)}{\tau_j} = -\frac{\widehat{A}(t,\tau_j)}{\tau_j} + \frac{B(\tau_j)}{\tau_j} r_t + \frac{C(\tau_j)}{\tau_j} \pi_t + \frac{\widehat{D}(\tau)}{\tau} \eta_t + \epsilon_{\tau_j}(t). \quad (28)$$

The measurement errors, $\epsilon_{\tau_j}(t)$, are assumed to be serially and cross-sectionally uncorrelated, and uncorrelated with the innovations in the transition equations. The variance of these measurement errors is assumed to be of the form: $\sigma^2(\epsilon_{\tau_j}) = \sigma_b^2/\tau_j$ where σ_b is a parameter to be estimated. The final observation equation uses the realized rate of inflation, $\frac{P_t - P_{t-\Delta t}}{P_{t-\Delta t}}$,

$$\frac{P_t - P_{t-\Delta t}}{P_{t-\Delta t}} = \pi \Delta t + \epsilon_P(t). \tag{29}$$

The data set for the first approach consists of monthly returns on the value weighted market portfolio, and the returns and (estimated) book-to-market (B/M) ratios on four portfolios sorted according to the B/M ratio and firm size for the period from January 1950 to December 1999. Portfolios are formed at the beginning of July each year based on the B/M ratio at the end of the previous year and the firm size at the end of June. The portfolios are the Big and Small, Growth and Value portfolios which were constructed by Fama and French.[20] The Small (Big) firm portfolio contains the NYSE, AMEX and NASDAQ stocks with market equity below (above) the median of NYSE stocks in June. The Growth (Value) High (Low) B/M portfolios include the top (bottom) 30% of NYSE stocks ranked according to the B/M ratio at the end of December of the previous year. Sorting stocks according to both size and B/M yields four portfolios which we denote by VB, GB, VS, GS, where G, V, B, and S stand for Growth, Value, Big and Small, respectively.

Monthly values for the B/M ratio for each portfolio are constructed from the Fama-French annual book-to-market data by a two stage process. First, the book value is assumed to be constant during the year and the Book-to-Market ratio for the beginning of July (when the portfolio composition is revised) is calculated by taking the B/M ratio at the end of December of the previous year as reported by Fama and French and updating it by dividing by the cumulative returns from January to June. Then B/M ratios for August through December are calculated by dividing the previous month's ratio by the portfolio return for the month. The second stage is to adjust the monthly figures to smooth out the "splicing error" which appears as the new B/M ratio for the portfolio is reported at

[20]We are grateful to Eugene Fama and Ken French for providing us with these data.

the end of December. There are now two B/M ratios for each December, the one that is calculated by updating the previous year's value by the portfolio returns $(B/M)_{old}$, and the one that is calculated for the portfolio using the new balance sheet data, $(B/M)_{new}$. We replace the $(B/M)_{old}$ with $(B/M)_{new}$ for December, and update the B/M ratios for the previous eleven months by spreading the cumulative error linearly over this period.

The nominal risk free interest rate is approximated by the return on a one month Treasury Bill taken from CRSP, and the inflation rate is calculated from the Consumer Price Index. The realized real interest rate for each month is calculated by subtracting the realized rate of inflation from the riskless nominal return.

Table 1 reports summary statistics on the Market-to-Book ratios and returns for the four portfolios, as well as for the (realized) monthly real interest rate and market return. Both the mean and the variability of the Market-to-Book ratios are approximately five times as high for the two growth portfolios as for the value portfolios. The highest correlation is between the ratios of big and small value firms (0.98), while the lowest is between value small firms and growth big firms (0.87). The correlations for the returns are generally lower, with the highest for value and growth small firms and the lowest for small growth firms and big value firms. The big growth firm portfolio has the highest correlation with the market return (0.97).

State variable instruments are constructed by taking the log of the ratios of the portfolio Book-to-Market ratios which are defined as $P_{GV}^{B} \equiv \ln\left[\frac{(B/M)_{VB}}{(B/M)_{GB}}\right]$, $P_{GV}^{S} \equiv \ln\left[\frac{(B/M)_{VS}}{(B/M)_{GS}}\right]$, $P_{BS}^{V} \equiv \ln\left[\frac{(B/M)_{VS}}{(B/M)_{VB}}\right]$, and $P_{BS}^{G} \equiv \ln\left[\frac{(B/M)_{GS}}{(B/M)_{GB}}\right]$, where P_{GV}^{B} is the (log scaled) price ratio of growth to value for big firms; P_{GV}^{S} is the corresponding ratio for small firms, P_{BS}^{V} is the ratio of prices of big to small (value) firms, and P_{BS}^{G} is the ratio of prices of big to small (growth) firms. The fifth state variable proxy is TS, the term spread between the yields on the 10 and 1 year Treasury Bonds, taken from CRSP. Only three of the four log price ratios are independent; therefore P_{BS}^{G} is omitted from the regressions that are reported below. Table 2 reports summary statistics for the state variable proxies, and

Figure 1 plots the time series of the proxies. The correlations between the state variable proxies are low, suggesting that all four proxies may be useful in predicting the investment opportunity set. The variability of the (scaled) price ratio between growth and value firms is 50% higher for large firms than for small firms, and three times as large as the price ratio between big and small value firms. Not surprisingly, all four state variable proxies are highly autocorrelated. The augmented Dickey-Fuller statistics reported in Table 2 strongly reject the null hypothesis of non-stationarity for three out of the four variables. Figure 1 shows that the growth-value ratios for big and small firms diverge for long periods of time while their mean values are quite close.

The data set for the second approach consists of monthly data on inflation and yields on eight constant maturity zero coupon U.S. treasury bonds with maturities of 3, 6 months, and 1, 2, 3, 4, 5, and 10 years for the period from March 1950 to September 1996.[21] Table 1 reports summary statistics for the bond yield data. The sample mean of the bond yields increases slightly with maturity, while the standard deviation remains relatively constant across maturities. The inflation rate during the same sample period is calculated from the CPI and has a sample mean of 4.1% and a sample standard deviation of 1.2%.

5 Empirical Results

In Section 5.1 we show that time variation in investment opportunities is captured by the scaled price ratios whose innovations correspond to the returns on the Fama-French hedge portfolios. We then estimate the investment opportunity set state variables, (r, η), as the fitted variables from the regression of normalized stock and T-bill returns on the log price ratios; these fitted values are referred to as the 'return-based' estimates of the state variables. In Section 5.2 we show that the time series of nominal bond yields and inflation

[21]We thank Luis Viceira for providing the data.

also provide strong evidence of time variation in the real interest rate and the Sharpe ratio; the filtered estimates of the state variables obtained from the yield data are referred to as the ‘yield-based’ estimates. Finally in Section 5.3, we show that the yield-based estimates are reflected in the prices of the Fama-French portfolios, that innovations in the yield-based estimates are correlated with the returns on the Fama-French hedge portfolios, and that the yield-based estimates are correlated with the return-based estimates. All of these evidence suggests an integrated bond-stock market in which the Fama-French hedge portfolio returns are correlated with innovations in the state variables that describe the investment opportunity set. This is consistent with the ICAPM rationale for the empirical success of the Fama-French three factor model.

5.1 Return-based Estimates of the State Variables

We assume initially that the market volatility, σ_M, is a constant, so that the Sharpe ratio, η, is proportional to $\mu_M - r$, the market risk premium. Then, in order to determine whether the proposed state variable proxies (the log value ratios and the term spread) have predictive power for the investment opportunity set variables (r, η), the market excess return and real interest rate were regressed on the state variable proxies in OLS regressions, using the Newey-West (1987) adjustment for heteroscedasticity and autocorrelation to compute the standard errors. The regressions were estimated both with and without the term spread variable, TS, because, while the model of Section 2 suggests that TS is a useful state variable proxy, it does not correspond to any of the Fama-French portfolios.

As predicted, the proposed state variable proxies have significant predictive power for both the market excess return and the real interest rate. Both P^S_{GV} and P^V_{BS} are marginally significant in predicting the market excess return, and all three state variable proxies are significant in the regression for the real interest rate. When TS is included in the regression both TS and P^S_{GV} are significant in the market excess return regression, while in the real interest rate regression TS is not significant but all three state variable proxies remain significant. These regression results provide strong evidence that the portfolio price ratios whose innovations correspond to the returns on the Fama-French SMB and HML portfolios, along with the term spread, are state variables in the Merton (1973) sense in that they predict the instantaneous investment opportunity set.

To this point we have assumed that the volatility of the market return is constant, so that the expected market excess return is proportional to the Sharpe ratio which, together with the real interest rate, are the sufficient statistics for the current investment opportunity set. Since there is evidence that market volatility is not constant,[22] rows 1 and 2 of Table 4 report estimates of the market excess return equation using an E-GARCH specification. As in the OLS regression that includes the term spread, P^S_{GV} is significant in predicting the market excess return. There is also strong evidence of GARCH effects, and since all four state variable proxies are highly significant in the market volatility equation, it is still possible that they affect the Sharpe ratio.

The GARCH results suggest that the market excess return may be a poor measure of the Sharpe ratio since the market volatility is varying. Therefore, SH, an estimate of the Sharpe ratio, was calculated for each month by taking the ratio of the market excess return in that month to the corresponding market volatility estimated using the full GARCH model with the term spread variable, row 2 of Table 4. The E-GARCH and OLS regressions were then repeated using SH in place of the market excess return.

Table 3 also reports the results of the OLS regression in which SH is the dependent variable. The results are qualitatively similar to those obtained when r_M is the dependent variable. Only P^S_{GV} and TS are significant in predicting SH when TS is included, and none of the state variable proxies is significant when TS is omitted.

Rows 3 and 4 of Table 4 report the E-GARCH estimates that are obtained using SH as the dependent variable. As we would expect, since the dependent variable is now a

[22] See French *et al. (1987).*

standardized variable, there is little evidence of GARCH effects, so that the estimates are similar to the OLS estimates in Table 3: P^S_{GV} and TS continue to be significant in predicting SH.

In summary, the OLS and E-GARCH estimates confirm the significance of P^S_{GV} and TS for predicting SH. The OLS results show that all three price-ratio state variable proxies are highly significant in predicting the real interest rate, while TS is not significant. These results provide empirical support for an ICAPM-based rationale for the Fama-French finding that covariances with returns on the HML and SMB portfolios are priced. They also suggest that covariances with a portfolio whose returns are correlated with innovations in the term spread will also be priced.[23] Finally, the time series of filtered values of the real interest rate and Sharpe ratio from the OLS regressions including TS were calculated. These are referred to as the 'return-based' estimates of the state variables since they are derived directly from the real T-bill return and market excess return. The parameters of the joint stochastic process of r and η that were estimated using the filtered values of the return-based estimates are reported in Row 1 of Table 5 and will be discussed in Section 5.3.

5.2 Yield-based Estimates of the State Variables

In this section we report the results of using a Kalman filter to estimate the dynamics of the state variables, r and η, from data on nominal bond yields and inflation. In order to identify the process for the Sharpe ratio, η, it is necessary to impose a restriction that determines the overall favorableness of investment opportunities.[24] For purposes of identification we set $\bar{\eta}$ equal to 0.7, which is approximately the value obtained by estimating equation (5.3) using the time series of return-based estimate η obtained in

[23] Chen, Roll and Ross (1986).

[24] Equation (4) shows that the structure of risk premia is invariant up to a scalar multiplication of η and the vector of inverse security correlations with the pricing kernel with typical element, $1/\rho_{Vm}$.

Section 5.1.[25] To improve the efficiency of estimation, the long run means for r and π were set equal to the corresponding historical mean values. The standard errors of all other parameters are understated because the standard errors of $\bar{r}$ and $\bar{\pi}$ are ignored. The results are reported in Row 2 of Table 5.

The variance of the measurement error in yield was assumed to be inversely proportional to the maturity τ. The estimate of the measurement error parameter, σ_b, implies that the standard deviation of the measurement error varies from 25 basis points for the three month maturity to 4 basis points for the ten year maturity, so that the model fits the yield data quite well. The volatilities of unexpected and expected inflation, σ_P and σ_π, are around 4% and 1% per year respectively, while the sample unconditional standard deviation of realized inflation is only about 1.2% per year. The standard deviation of the estimated real interest rate process, σ_r, is 2.1% per year, so that the real interest rate is much more volatile than the expected inflation rate. The estimated mean reversion intensity for the interest rate, κ_r, is 0.14 per year which implies a half life of about 5 years. The expected rate of inflation rate follows almost a random walk. The volatility of the Sharpe ratio process, σ_η, is 0.1 per year which compares with the imposed long run mean value of 0.7. The Wald statistic which tests the null hypothesis that $\sigma_\eta = \kappa_\eta = 0$ (so that η is a constant) is highly significant so, given the pricing model, there is strong evidence from the bond yield data that the Sharpe ratio is time varying. Moreover, the t-statistics on $\rho_{\eta m}$ and ρ_{rm} strongly reject the null that the opportunity set state variables, r and η, are unpriced, providing strong evidence in favor of the ICAPM.

[25] Mackinlay (1995) reports an average Sharpe ratio of around 0.40 for the $S\&P500$ for the period 1981-1992.

5.3 Evaluation of the State Variable Estimates

In the absence of model and estimation error, we should expect the yield-based estimates of (r, η) to be identical to the return-based estimates. Therefore in this section we compare the two sets of state variable estimates, and examine whether the yield-based estimates are reflected in the prices of the Fama-French portfolios, and whether innovations in the yield-based state variable estimates are correlated with the returns on the Fama-French hedge portfolios.

Figure 2 plots the time series of the yield-based and return-based real interest rate estimates. There is only limited correspondence between the two series, the yield-based estimate being much more variable than the return-based estimate. The correlation between the levels of the two estimated series is -0.08, which is not supportive of our conjecture; however the correlation between the monthly innovations in the series is slightly positive (0.08). Figure 3 plots the time series of the yield-based and return-based Sharpe ratio estimates, η; for this variable, the correspondence between the return and yield based estimates is much stronger. The correlation between the levels of the two estimated series is 0.47, and the correlation between the monthly innovations in the series is 0.24. Table 6, which reports the results of simple regressions of the return-based estimates on the yield-based estimates, confirms that the relation between the two Sharpe ratio (η) estimates is much stronger than that between the two real interest rate (r) estimates. In view of the radically different approaches and data sets (bond *yields* on the one hand, and equity and bill *returns* on the other) used to generate these two sets of estimates, the correspondence between them is highly encouraging.

The shaded areas in the figures correspond to periods of U.S. recession as determined by the National Bureau of Economic Research.[26] For both sets of estimates, the correlation between r and η is about -0.36, and recessions are generally associated with a declining

[26]The period of recession is measured from peak to trough.

real interest rate but increasing Sharpe Ratio. Whitelaw (1997) and Perez-Quiros and Timmermann (2000) have found similar cyclical patterns in the Sharpe ratio. Their approach to estimate the Sharpe ratio is similar to that employed to obtain our return-based estimates except that, instead of using Fama-French portfolio price ratios as predictors, they use the more conventional dividend yield, default spread, and yield variables.

The return-based estimates for the Sharpe ratio, η, were used to estimate the parameters of the Ornstein-Uhlenbeck processes for the state variables by non-linear least squares. The results are reported in Row 1 Table 5 along with the corresponding parameter estimates from the bond yield data using the Kalman filter (Row 2 Table 5). The estimated mean reversion coefficients for the Sharpe ratio and the real interest rate using the return-based estimates of η and r are about 1.2 and 0.5: the corresponding half-lives are about 0.6 and 1.44 years. In contrast, for the much smoother series of the Sharpe ratio and the real interest rate obtained from the bond yield data, the mean reversion coefficients are 0.22 and 0.14, implying half-lives of 3.2 and 5 years. The estimated long term (annualized) Sharpe ratio is about 0.70 for the return-based estimates.[27]

As a further check on the relation between the yield-based estimates of the state variables and the FF hedge portfolios, the log portfolio price ratios were regressed on the yield-based estimates of r, η and π, and the results are reported in Panel A of Table 7. All four price ratios are negatively related to the estimated real interest rate. For the "growth-value" ratios this is consistent with Lemma 1: high real interest rates depress the prices of growth firms relative to value firms. High real interest rates also appear to depress the prices of big firms relative to small firms. The Sharpe ratio has a significant positive effect on the "growth-value" price ratio for small firms, and significant negative effects on the "big-small" price ratio for growth firms, and the "growth-value" price ratio for big firms. The direction of the effect implied by the model developed in Section 2 depends on both the relative durations of the firm cash flows and their correlations with

[27] For the bond yield estimates $\bar{\eta}$ was set at 0.7.

the pricing kernel, ρ_{Ym}. Expected inflation also has a significant effect on two of the price ratios which suggests that this variable has information about the relative real cash flows of the different portfolios. In summary, there is strong evidence that the portfolio price ratios, which are the basis of the Fama-French hedge portfolio returns, are related to the state variables which determine the investment opportunity set.

If the ICAPM is to provide an explanation for the risk premia on the FF hedge portfolios it must be the case that innovations in the investment opportunity set state variables are correlated with the returns on these portfolios. We have already seen evidence of this in that the equity market state variable proxies (the price ratios of the portfolios corresponding to the FF hedge portfolio returns) do have predictive power for the investment opportunity set. As a further test of this we examine whether the innovations in the yield-based state variable estimates are correlated with the returns on the FF portfolios.[28] To this end we calculate the innovations, $\hat{\epsilon_r}$, $\hat{\epsilon_\pi}$ and $\hat{\epsilon_\eta}$, in the yield-based state variable estimates using the parameter values reported in Table 5, and then regress these estimated innovations on the returns on the three FF portfolios. The results are reported in Panel B of Table 7. The innovation in r is significantly related to the excess return on the market portfolio and to the return on the HML portfolio. The innovation in η is significantly related to the returns on both the SMB and HML portfolios.[29] Once again, these results point to a link between the FF portfolio returns and innovations in the investment opportunity set. Given the ICAPM, we should expect to find risk premia associated with loadings on these portfolio returns.

[28]Since the return-based state variable estimates are linear functions of the price ratios, their innovations are correlated with the returns on the FF portfolios by construction.

[29]The innovation in expected inflation is significantly negatively related to the return on the HML portfolio.

6 Implications for Stock Price and Volatility

Cochrane (1991) has pointed out that time variation in the investment opportunities can substantially increase the volatility of stock prices relative to a constant discount rate model. To assess the effect on stock price volatility of variation in the state variables, η and r, the valuation factor $v(r, \eta, \tau)$ was calculated for each month from March 1950 to September 1996 using the yield-based estimates of r and η, and the parameters of their joint stochastic process (5). The cash flow expectation was assumed to have a "unit beta" so that $\beta_{YM} \equiv \sigma_{YM}/\sigma_M^2 = 1$, where M denotes the CRSP value weighted market portfolio and σ_M was taken as 14.5%. In order to assess the importance of intertemporal asset pricing considerations, two different asset pricing models were used to calculate the valuation factor. The first is the single period CAPM which attributes no risk premium to state variable innovations so that the process for the pricing kernel is $dm/m = -rdt - \eta dz_M$. The second asset pricing model is the ICAPM which allows for risk premia associated with the state variable innovations so that the pricing kernel is $dm/m = -rdt - \eta(\zeta_M dz_M - \zeta_\eta dz_\eta - \zeta_r dz_r)$. For both the CAPM and the ICAPM, the correlations between the innovations in r and η and the market return, ρ_{Mr} and $\rho_{M\eta}$, were taken as the sample estimates, -0.12 and 0.01 respectively. For the ICAPM, the coefficients of the pricing kernel were chosen to match the yield-based estimates of the parameters $\rho_{mr}(-0.82)$ and $\rho_{m\eta}(-0.39)$ and to satisfy the constraint that $\mathrm{Var}\,(dm/m) = \eta^2$.

In order to analyze the separate effects of variation in r and η on the variability of the valuation factor, three time series were calculated for both asset pricing model assumptions: $v^\eta(\eta, \tau) \equiv e^{A(\tau) - D(\tau)\eta_t}$, $v^r(r, \tau) \equiv e^{A(\tau) - B(\tau) r_t}$, and $v(r, \eta, \tau) \equiv e^{A(\tau) - B(\tau) r_t - D(\tau)\eta_t}$. Figure 4 plots the three time series for the two model assumptions when $\tau = 10$ years. The qualitative behavior of v under the CAPM and ICAPM assumptions are similar. Figure 4 shows that under both assumptions valuations display strong cyclical variations, rising before business cycle peaks and falling during recessions. Moreover, most of this variation is due to the variation in the Sharpe ratio rather than the real interest rate.

Table 8, which provides summary statistics on these time series for three different maturies, shows that under both the CAPM and ICAPM assumptions the variability of the valuation factor is strongly increasing in maturity. The effect of the ICAPM assumption is to significantly decrease the average value of the valuation factor at long maturities relative to the CAPM due to the additional risk premia associated with r and η. At long maturities, the ICAPM also increases the component of return volatility that is associated with η and therefore the total return volatility. Under the CAPM assumption, the effects of variation in r and η on the variation of dv/v are of similar magnitude while under the ICAPM assumption the effect of variation in η predominates, especially at long maturities.[30] Campbell and Ammer (1993) find that variation in real interest rates accounts for only a minor component of the variation in stock returns. We also find that the effect of variation in r is negligible as compared to the effect of variation in η when the valuation factor is calculated using return-based estimates of r and η.[31]

In summary, the yield-based parameter estimates suggest that the variation in the investment opportunities accounts for a significant proportion of stock price variation and that variation in the price of risk is more important than variation in the interest rate.

7 Conclusion

In this paper we have developed a simple model of asset valuation for a setting in which real interest rates and risk premia vary stochastically. The model implies, first, that the ratios of the prices of Fama-French size and value portfolios, as well as the

[30] The effects of r and η variation are not additive because of the negative correlation between these two variables.

[31] The return-based estimation is more similar to that of Campbell and Ammer (1993) in that it relies on linear regression of returns on predictive variables. This procedure yields much larger estimates of the mean-reversion intensities for r and η shown in Table 5.

term spreads, will carry information about the real interest rate and the Sharpe ratio. This provides a justification in the context of ICAPM for the risk premia that Fama and French have found to be associated with the HML and SMB portfolio returns. We find strong empirical evidence that the FF portfolios do predict the real interest rate and the Sharpe ratio. The model also implies that zero-coupon bond yields are linearly related to the state variables r and η, and data on bond yields are used to provide a second set of estimates of the state variables. We find that these yield-based estimates are related to the first set of estimates, as well as to the returns on the Fama-French portfolios. Both sets of estimates of the Sharpe ratio display strong cyclical variation, rising during recessions and falling during expansions. The yield-based estimates were used to calculate time series of valuation factors. Under both the CAPM and ICAPM assumptions, the valuation factors reflect the cyclical behavior of the state variable estimates and suggest that a significant variation in stock prices is due to variations in discount rates rather than variation in cash flow expectations. Finally, the results suggest that a simple CAPM may significantly overstate the valuation of long maturity claims if these claim values are determined by an intertemporal version of the CAPM.

Appendix

A. Proof of Theorems 1 and 2

The real part of the economy is described by the processes for the real pricing kernel, the real interest rate, and the maximum Sharpe ratio (5.1)-(5.3), while the nominal part of the economy is described by the processes for the price level and the expected inflation rate (15)-(16). Under the risk neutral probability measure Q, we can write these processes as:

$$\begin{aligned} dr &= \kappa_r(\bar{r} - r)dt - \sigma_r \rho_{mr} \eta dt + \sigma_r dz_r^Q \qquad \text{(A1)} \\ d\pi &= \kappa_\pi(\bar{\pi} - \pi)dt - \sigma_\pi \rho_{m\pi} \eta dt + \sigma_\pi dz_\pi^Q \qquad \text{(A2)} \\ d\eta &= \kappa_\eta^*(\bar{\eta}^* - \eta)dt + \sigma_\eta dz_\eta^Q \qquad \text{(A3)} \end{aligned}$$

where $\kappa_\eta^* = \kappa_\eta + \sigma_\eta \rho_{m\eta}$ and $\bar{\eta}^* = \frac{\kappa_\eta \bar{\eta}}{\kappa_\eta^*}$.

Let Y, whose stochastic process is given by (6), denote the expectation of a nominal cash flow at a future date T, X_T. The process for $\xi \equiv Y/P$, the deflated expectation of the nominal cash flow, under the risk neutral probability measure can be written as:

$$\frac{d\xi}{\xi} = \left[-\pi - \sigma_Y \sigma_P \rho_{YP} + \sigma_P^2 - \eta(\sigma_Y \rho_{Ym} - \sigma_P \rho_{Pm})\right] dt + \sigma_Y dz_Y^Q - \sigma_P dz_P^Q. \qquad \text{(A4)}$$

The real value at time t of the claim to the nominal cash flow at time T, X_T, is given by expected discounted value of the real cash flow under Q:

$$\begin{aligned} V(\xi, r, \pi, \eta, T-t) &= \mathrm{E}_t^Q\left[\frac{X_T}{P_T} \exp^{-\int_t^T r(s)ds}\right] = \mathrm{E}_t^Q\left[\frac{Y_T}{P_T} \exp^{-\int_t^T r(s)ds}\right] \\ &= \mathrm{E}_t^Q\left[\xi_T \exp^{-\int_t^T r(s)ds}\right] \qquad \text{(A5)} \end{aligned}$$

Using equation (A4), we have

$$\begin{aligned}\xi_T &= \xi_t \exp\left\{\left(-\frac{1}{2}\sigma_Y^2+\frac{1}{2}\sigma_P^2\right)(T-t)-(\sigma_Y\rho_{Ym}-\sigma_P\rho_{Pm})\int_t^T \eta(s)ds\right.\\ &- \left.\int_t^T \pi(s)ds+\sigma_Y\int_t^T dz_Y^Q-\sigma_P\int_t^T dz_P^Q\right\}. \qquad \text{(A6)}\end{aligned}$$

A tedious calculation from equations (A1), (A2), and (A3) gives us the following results:

$$\begin{aligned}\int_t^T \eta(s)ds &= \eta_t\frac{1-e^{-\kappa_\eta^*(T-t)}}{\kappa_\eta^*}+\bar{\eta}^*\left[T-t-\frac{1-e^{-\kappa_\eta^*(T-t)}}{\kappa_\eta^*}\right]\\ &+ \sigma_\eta\int_t^T \frac{1-e^{-\kappa_\eta^*(T-s)}}{\kappa_\eta^*}dz_\eta^Q(s), \qquad \text{(A7)}\end{aligned}$$

$$\begin{aligned}\int_t^T \pi(s)ds &= \pi_t\frac{1-e^{-\kappa_\pi(T-t)}}{\kappa_\pi}+\left(\bar{\pi}-\frac{\sigma_\pi\rho_{m\pi}\bar{\eta}^*}{\kappa_\pi}\right)\left[T-t-\frac{1-e^{-\kappa_\pi(T-t)}}{\kappa_\pi}\right]\\ &+ \left(\frac{\sigma_\pi\rho_{m\pi}\eta_t}{\kappa_\eta^*-\kappa_\pi}-\frac{\sigma_\pi\rho_{m\pi}\bar{\eta}^*}{\kappa_\eta^*-\kappa_\pi}\right)\left[\frac{1-e^{-\kappa_\eta^*(T-t)}}{\kappa_\eta^*}-\frac{1-e^{-\kappa_\pi(T-t)}}{\kappa_\pi}\right]\\ &+ \frac{\sigma_\pi\rho_{m\pi}\sigma_\eta}{\kappa_\eta^*-\kappa_\pi}\int_t^T\left[\frac{1-e^{-\kappa_\eta^*(T-s)}}{\kappa_\eta^*}-\frac{1-e^{-\kappa_\pi(T-s)}}{\kappa_\pi}\right]dz_\eta^Q(s)\\ &+ \sigma_\pi\int_t^T \frac{1-e^{-\kappa_\pi(T-s)}}{\kappa_\pi}dz_\pi^Q(s) \qquad \text{(A8)}\end{aligned}$$

and

$$
\begin{aligned}
\int_t^T r(s)ds &= r_t\frac{1-e^{-\kappa_r(T-t)}}{\kappa_r} + \left(\bar{r} - \frac{\sigma_r\rho_{mr}\bar{\eta}^*}{\kappa_r}\right)\left[T - t - \frac{1-e^{-\kappa_r(T-t)}}{\kappa_r}\right] \\
&+ \left(\frac{\sigma_r\rho_{mr}\eta_t}{\kappa_\eta^* - \kappa_r} - \frac{\sigma_r\rho_{mr}\bar{\eta}^*}{\kappa_\eta^* - \kappa_r}\right)\left[\frac{1-e^{-\kappa_\eta^*(T-t)}}{\kappa_\eta^*} - \frac{1-e^{-\kappa_r(T-t)}}{\kappa_r}\right] \\
&+ \frac{\sigma_r\rho_{mr}\sigma_\eta}{\kappa_\eta^* - \kappa_r}\int_t^T\left[\frac{1-e^{-\kappa_\eta^*(T-s)}}{\kappa_\eta^*} - \frac{1-e^{-\kappa_r(T-s)}}{\kappa_r}\right]dz_\eta^Q(s) \\
&+ \sigma_r\int_t^T\frac{1-e^{-\kappa_r(T-s)}}{\kappa_r}dz_r^Q(s) \qquad \text{(A9)}
\end{aligned}
$$

Substituting equations (A6)-(A9) into equation (A5) yields

$$
V(\xi, r, \pi, \eta, T-t) = \xi_t G \mathrm{E}_t^Q\left[\exp^{\psi}\right], \qquad \text{(A10)}
$$

where G is given by

$$
G = \exp\left\{E(\tau) - B(\tau)r_t - C(\tau)\pi_t - D(\tau)\eta_t\right\} \qquad \text{(A11)}
$$

and

$$
\begin{aligned}
B(\tau) &= \frac{1-e^{-\kappa_r(T-t)}}{\kappa_r} \qquad \text{(A12)} \\
C(\tau) &= \frac{1-e^{-\kappa_\pi(T-t)}}{\kappa_\pi} \qquad \text{(A13)} \\
D(\tau) &= d_1 + d_2e^{-\kappa_\eta^*\tau} + d_3e^{-\kappa_r\tau} + d_4e^{-\kappa_\pi\tau} \qquad \text{(A14)} \\
E(\tau) &= \left(-\frac{1}{2}\sigma_Y^2 + \frac{1}{2}\sigma_P^2 - \bar{r} - \bar{\pi} - d_1\kappa_\eta^*\bar{\eta}^*\right)\tau + \left(\bar{r} - d_3\kappa_\eta^*\bar{\eta}^*\right)B(\tau) \\
&+ \left(\bar{\pi} - d_4\kappa_\eta^*\bar{\eta}^*\right)C(\tau) - d_2\kappa_\eta^*\bar{\eta}^*d(\tau) \qquad \text{(A15)}
\end{aligned}
$$

with $d(\tau) = \left(1 - e^{-\kappa_\eta^*(T-t)}\right)/\kappa_\eta^*$, and finally

$$
\begin{aligned}
d_1 &= -\frac{\sigma_P \rho_{mP} - \sigma_Y \rho_{mY}}{\kappa_\eta^*} - \frac{\sigma_r \rho_{mr}}{\kappa_r \ \kappa_\eta^*} - \frac{\sigma_\pi \rho_{m\pi}}{\kappa_\pi \kappa_\eta^*} && \text{(A16)} \\
d_2 &= -\frac{\sigma_Y \rho_{mY}}{\kappa_\eta^*} - \frac{\sigma_r \rho_{mr}}{(\kappa_\eta^* - \kappa_r)\kappa_\eta^*} - \frac{\sigma_\pi \rho_{m\pi}}{(\kappa_\eta^* - \kappa_\pi)\kappa_\eta^*} && \text{(A17)} \\
&= -d_1 - d_3 - d_4 \\
d_3 &= \frac{\sigma_r \rho_{mr}}{(\kappa_\eta^* - \kappa_r)\kappa_r} && \text{(A18)} \\
d_4 &= \frac{\sigma_\pi \rho_{m\pi}}{(\kappa_\eta^* - \kappa_r)\kappa_\pi} && \text{(A19)}
\end{aligned}
$$

The stochastic variable ψ is a linear function of the Brownian motions:

$$
\begin{aligned}
\psi &= \sigma_\eta \int_t^T \left[d_2\left(1 - e^{-\kappa_\eta^*(T-s)}\right) + d_3\left(1 - e^{-\kappa_r(T-s)}\right) + d_4\left(1 - e^{-\kappa_\pi(T-s)}\right)\right] dz_\eta^*(s) \\
&- \frac{\sigma_r}{\kappa_r} \int_t^T \left(1 - e^{-\kappa_r(T-s)}\right) dz_r^*(s) - \frac{\sigma_\pi}{\kappa_\pi} \int_t^T \left(1 - e^{-\kappa_\pi(T-s)}\right) dz_\pi^*(s) \\
&+ \sigma_Y \int_t^T dz_Y^*(s) - \sigma_P \int_t^T dz_P^*(s). && \text{(A20)}
\end{aligned}
$$

Since ψ is normally distributed with mean zero, V is given by

$$
V(\xi, r, \pi, \eta, T-t) = \xi_t G_1 \exp\left\{\frac{1}{2}\mathrm{Var}_t(\psi)\right\} \qquad \text{(A21)}
$$

Calculating $\mathrm{Var}_t(\psi)$ and collecting terms, we get that

$$
V(\xi, r, \pi, \eta, T-t) = \xi_t \exp\left\{A(\tau) - B(\tau) r_t - C(\tau)\pi_t - D(\tau)\eta_t\right\} \qquad \text{(A22)}
$$

where

$$A(\tau) = a_1\tau + a_2\frac{1-e^{-\kappa_r\tau}}{\kappa_r} + a_3\frac{1-e^{-\kappa_\pi\tau}}{\kappa_\pi} + a_4\frac{1-e^{-\kappa_\eta^*\tau}}{\kappa_\eta^*}$$
$$+a_5\frac{1-e^{-2\kappa_r\tau}}{2\kappa_r} + a_6\frac{1-e^{-2\kappa_\pi\tau}}{2\kappa_\pi} + a_7\frac{1-e^{-2\kappa_\eta^*\tau}}{2\kappa_\eta^*}$$
$$+a_8\frac{1-e^{-(\kappa_\eta^*+\kappa_r)\tau}}{\kappa_\eta^*+\kappa_r} + a_9\frac{1-e^{-(\kappa_\eta^*+\kappa_\pi)\tau}}{\kappa_\eta^*+\kappa_\pi} + a_{10}\frac{1-e^{-(\kappa_r+\kappa_\pi)\tau}}{\kappa_r+\kappa_\pi}. \quad \text{(A23)}$$

Define $a_0 \equiv \frac{\sigma_{r\eta}}{\kappa_r} + \frac{\sigma_{\pi\eta}}{\kappa_\pi} + \sigma_{P\eta} - \sigma_{Y\eta} - \kappa_\eta^*\bar{\eta}^*$, $\bar{r}^* \equiv \bar{r} - \frac{\sigma_{Pr}-\sigma_{Yr}}{\kappa_r}$, and $\bar{\pi}^* \equiv \bar{\pi} - \frac{\sigma_{P\pi}-\sigma_{Y\pi}}{\kappa_\pi}$, then $a_1, \ldots, a_{10}$ are expressed as

$$a_1 = \sigma_P^2 - \sigma_{YP} + \frac{\sigma_r^2}{2\kappa_r^2} + \frac{\sigma_\pi^2}{2\kappa_\pi^2} + \frac{\sigma_{r\pi}}{\kappa_r\kappa_\pi} + \frac{\sigma_\eta^2}{2}d_1^2 - \bar{r}^* - \bar{\pi}^* + a_0 d_1 \quad \text{(A24)}$$
$$a_2 = \bar{r}^* - \frac{\sigma_r^2}{\kappa_r^2} - \frac{\sigma_{r\pi}}{\kappa_r\kappa_\pi} - \frac{\sigma_{r\eta}}{\kappa_r}d_1 + a_0 d_3 + \sigma_\eta^2 d_1 d_3 \quad \text{(A25)}$$
$$a_3 = \bar{\pi}^* - \frac{\sigma_\pi^2}{\kappa_\pi^2} - \frac{\sigma_{r\pi}}{\kappa_r\kappa_\pi} - \frac{\sigma_{\pi\eta}}{\kappa_\pi}d_1 + a_0 d_4 + \sigma_\eta^2 d_1 d_4 \quad \text{(A26)}$$
$$a_4 = a_0 d_2 + \sigma_\eta^2 d_1 d_2 \quad \text{(A27)}$$
$$a_5 = \frac{\sigma_r^2}{2\kappa_r^2} + \frac{\sigma_\eta^2}{2}d_3^2 - \frac{\sigma_{r\eta}}{\kappa_r}d_3 \quad \text{(A28)}$$
$$a_6 = \frac{\sigma_\pi^2}{2\kappa_\pi^2} + \frac{\sigma_\eta^2}{2}d_4^2 - \frac{\sigma_{\pi\eta}}{\kappa_\pi}d_4 \quad \text{(A29)}$$
$$a_7 = \frac{\sigma_\eta^2}{2}d_2^2 \quad \text{(A30)}$$
$$a_8 = -\frac{\sigma_{r\eta}}{\kappa_r}d_2 + \sigma_\eta^2 d_2 d_3 \quad \text{(A31)}$$
$$a_9 = -\frac{\sigma_{\pi\eta}}{\kappa_\pi}d_2 + \sigma_\eta^2 d_2 d_4 \quad \text{(A32)}$$
$$a_{10} = \frac{\sigma_{r\pi}}{\kappa_r\kappa_\pi} - \frac{\sigma_{\pi\eta}}{\kappa_\pi}d_3 - \frac{\sigma_{r\eta}}{\kappa_r}d_4 + \sigma_\eta^2 d_3 d_4 \quad \text{(A33)}$$

Theorems 1 and 2 follow as special cases of equation (A22). Theorem 1 is obtained by setting σ_P and the parameters in the expected inflation process (A2) to zero. Theorem 2 is obtained by setting σ_Y to zero.

B. Proof of Lemmas 1 and 2

Proof of Lemma 1:

It is immediate that $B(\tau)$, $\frac{\partial B(\tau)}{\partial \tau} > 0$. From equations (A14) and (A16-A19), we note that

$$D(0) \;=\; d_1 + d_2 + d_3 = 0$$

Taking the derivative of (A14) with respect to τ,

$$D_\tau \;=\; \sigma_Y \rho_{Ym} e^{-\kappa_\eta^* \tau} + \frac{\sigma_r \rho_{mr}}{\kappa_\eta^* - \kappa_r}(e^{-\kappa_\eta^* \tau} - e^{-\kappa_r \tau}). \tag{B1}$$

Then the assumptions $\rho_{Ym} > 0$ and $\rho_{mr} < 0$ imply that $D_\tau \geq 0$, so that $D(\tau) \geq 0\ \forall \tau$.

Proof of Lemma 2:

Applying Ito's lemma to the V function from Theorem 1, the expected return on the claim can be written as:

$$\begin{aligned}
\mu \equiv \mu(r, \eta, \tau) \;&=\; -A_\tau + rB_\tau + \eta D_\tau + \frac{1}{2}D^2\sigma_\eta^2 + BD\rho_{\eta r}\sigma_\eta\sigma_r + \frac{1}{2}B^2\sigma_r^2 \\
&\quad -D\rho_{Y\eta}\sigma_Y\sigma_\eta - B\rho_{Yr}\sigma_Y\sigma_r - D\kappa_\eta(\bar{\eta} - \eta) - B\kappa_r(\bar{r} - r) \\
&=\; D_\tau\eta + B_\tau r + D\kappa_\eta\eta + B\kappa_r r \\
&=\; r + (D_\tau + D\kappa_\eta)\eta, \qquad\qquad \text{(B2)}
\end{aligned}$$

then

$$\begin{aligned}
\frac{\partial \mu}{\partial \tau} \;&=\; (D_{\tau\tau} + D_\tau\kappa_\eta)\eta \\
&=\; \eta\left[\sigma_Y\rho_{Ym}\kappa_\eta - \sigma_r\rho_{mr}e^{-\kappa_r\tau} + \frac{\sigma_r\rho_{mr}\sigma_\eta\rho_{m\eta}}{\kappa_r - \kappa_\eta^*}(e^{-\kappa_\eta^*\tau} - e^{-\kappa_r^*\tau})\right], \qquad \text{(B3)}
\end{aligned}$$

form which, it follows that $\frac{\partial \mu}{\partial \tau} \geq 0$ when $\rho_{mr}, \rho_{m\eta} \leq 0$ and $\rho_{Ym} \geq 0$.

Lemma 3 is straightforward and the proof is omitted.

C. Details of Kalman Filter

The yield-based estimates of the state variable dynamics are derived by applying a Kalman filter to data on bond yields and inflation using equations (28) and (29). The transition equations for the state variables, r, π and η are derived by discretizing equations (5.2), (5.3), and (16):

$$\begin{pmatrix} r_t \\ \pi_t \\ \eta_t \end{pmatrix} = \begin{pmatrix} e^{-\kappa_r \Delta t} & 0 & 0 \\ 0 & e^{-\kappa_\pi \Delta t} & 0 \\ 0 & 0 & e^{-\kappa_\eta \Delta t} \end{pmatrix} \begin{pmatrix} r_{t-\Delta t} \\ \pi_{t-\Delta t} \\ \eta_{t-\Delta t} \end{pmatrix} + \begin{pmatrix} \bar{r}\left[1 - e^{-\kappa_r \Delta t}\right] \\ \bar{\pi}\left[1 - e^{-\kappa_\pi \Delta t}\right] \\ \bar{\eta}\left[1 - e^{-\kappa_\eta \Delta t}\right] \end{pmatrix} + \begin{pmatrix} \epsilon_r(t) \\ \epsilon_\pi(t) \\ \epsilon_\eta(t) \end{pmatrix}, \tag{C1}$$

where the vector of innovations is related to the standard Brownian motions, dz_r, dz_π and dz_η, by

$$\begin{pmatrix} \epsilon_r(t) \\ \epsilon_\pi(t) \\ \epsilon_\eta(t) \end{pmatrix} = \begin{pmatrix} \sigma_r e^{-\kappa_r (t-\Delta t)} \int_{t-\Delta t}^{t} e^{-\kappa_r \tau} dz_r(\tau) \\ \sigma_\pi e^{-\kappa_\pi (t-\Delta t)} \int_{t-\Delta t}^{t} e^{-\kappa_\pi \tau} dz_\pi(\tau) \\ \sigma_\eta e^{-\kappa_\eta (t-\Delta t)} \int_{t-\Delta t}^{t} e^{-\kappa_\eta \tau} dz_\eta(\tau) \end{pmatrix}, \tag{C2}$$

and the variance-covariance matrix of the innovations is

$$Q = \begin{pmatrix} \frac{\sigma_r^2}{2\kappa_r}\left[1 - e^{-2\kappa_r \Delta t}\right] & \frac{\sigma_r \sigma_\pi \rho_{r\pi}}{\kappa_r + \kappa_\pi}\left[1 - e^{-(\kappa_r + \kappa_\pi)\Delta t}\right] & \frac{\sigma_r \sigma_\eta \rho_{r\eta}}{\kappa_r + \kappa_\eta}\left[1 - e^{-(\kappa_r + \kappa_\eta)\Delta t}\right] \\ \frac{\sigma_r \sigma_\pi \rho_{r\pi}}{\kappa_r + \kappa_\pi}\left[1 - e^{-(\kappa_r + \kappa_\pi)\Delta t}\right] & \frac{\sigma_\pi^2}{2\kappa_\pi}\left[1 - e^{-2\kappa_\pi \Delta t}\right] & \frac{\sigma_\pi \sigma_\eta \rho_{\pi\eta}}{\kappa_\pi + \kappa_\eta}\left[1 - e^{-(\kappa_\pi + \kappa_\eta)\Delta t}\right] \\ \frac{\sigma_r \sigma_\eta \rho_{r\eta}}{\kappa_r + \kappa_\eta}\left[1 - e^{-(\kappa_r + \kappa_\eta)\Delta t}\right] & \frac{\sigma_\pi \sigma_\eta \rho_{\pi\eta}}{\kappa_\pi + \kappa_\eta}\left[1 - e^{-(\kappa_\pi + \kappa_\eta)\Delta t}\right] & \frac{\sigma_\eta^2}{2\kappa_\eta}\left[1 - e^{-2\kappa_\eta \Delta t}\right] \end{pmatrix}. \tag{C3}$$

The first n observation equations assume that the observed yields at time t, $y_{\tau_j,t}$, on bonds with maturities τ_j, $j = 1, \cdots, n$, are given by equation (28) plus a measurement error terms, ϵ_{τ_j}:

$$y_{\tau_j,t} \equiv -\frac{\ln V(t, t+\tau_j)}{\tau_j} = -\frac{A(t,\tau_j)}{\tau_j} + \frac{B(\tau_j)}{\tau_j} r_t + \frac{C(\tau_j)}{\tau_j}\pi_t + \frac{D(\tau_j)}{\tau_j}\eta + \epsilon_{\tau_j}(t). \quad \text{(C4)}$$

The measurement errors, $\epsilon_{\tau_j}(t)$, are assumed to be serially and cross-sectionally uncorrelated and are uncorrelated with the innovations in the transition equations.

The final observation equation uses the realized rate of inflation:

$$\frac{P_t - P_{t-\Delta t}}{P_{t-\Delta t}} = \pi \Delta t + \epsilon_P(t), \quad \text{(C5)}$$

where $\epsilon_P = \sigma_P \int_{t-\Delta t}^{t} dz_P$ with variance $\sigma_P^2 \Delta t$, and is assumed to be uncorrelated with the yield measurement errors and the innovations in the transition equation.

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Table 1
Summary Statistics on Four Fama-French Portfolios and Bond Yields

Returns are in percent per month. r_f is the real riskless interest rate and r_M is the market excess returns. Portfolio GS is the Fama-French low book-to-market portfolio of small firms; VS the high book-to-market portfolio of small firms; GB the low book-to-market portfolio of big firms; VB the high book-to-market portfolio of big firms. The sample is from January 1950 to December 1999. The bond data are monthly constant maturity zero coupon U.S. Treasury yields for the period from March 1953 to September 1996. Inflation is calculated from the CPI data for the same sample period.

A.Distribution of Returns and Market to Book (M/B) ratios

	M/B				Return					
Portfolio	GS	VS	GB	VB	GS	VS	GB	VB	r_f	r_M
Mean	2.89	0.62	3.07	0.69	1.06	1.49	1.11	1.34	0.08	0.71
Minimum	1.15	0.25	1.47	0.28	-32.09	-27.98	-23.07	-18.95	-1.5	-22.83
Median	2.86	0.58	2.88	0.68	1.25	1.83	1.35	1.57	0.11	1.00
Maximum	5.82	1.20	6.28	1.18	24.94	29.60	21.43	20.95	1.12	16.00
Std.Dev	0.98	0.19	1.03	0.20	6.05	5.07	4.45	4.39	0.29	4.17
Autocorrelation	0.97	0.98	0.99	0.99	0.17	0.17	0.05	0.04	0.48	0.06

B. Correlations

Market to Book Ratios				
	GS	VS	GB	VB
GS	1			
VS	0.96	1		
GB	0.89	0.87	1	
VB	0.95	0.98	0.89	1

Portfolio and Market Returns					
	GS	VS	GB	VB	r_M
GS	1				
VS	0.89	1			
GB	0.83	0.74	1		
VB	0.75	0.88	0.78	1	
r_M	0.88	0.84	0.97	0.87	1

C. Bond Yields and Inflation (% per year)

Bond Maturity (years)	0.25	0.5	1	2	3	4	5	10	Inflation
Mean	5.35	5.57	5.77	5.99	6.13	6.24	6.32	6.54	4.10
Std. Dev.	3.03	3.07	3.07	3.03	3.00	2.97	2.96	2.91	1.16

Table 2
Summary Statistics for State Variable Proxies

P^B_{GV} is the (log of the) ratio of the market-to-book ratios for large growth and value firms; P^S_{GV} is the (log of the) ratio of the market-to-book ratios for small growth and value firms; P^V_{BS} is the (log of the) ratio of the market-to-book ratios for value big and small firms; TS is the term spread as measured by the difference between the yields on the 10 and 1 year Treasury Bonds (% per year). The sample period is from January 1950 to December 1999.

A.Distribution of log of Market to Book ratios and Term Spread				
	P^B_{GV}	P^S_{GV}	P^V_{BS}	TS
Mean	1.485	1.528	0.108	0.650
Minimum	1.157	1.270	-0.068	-3.856
Median	1.512	1.528	0.101	0.705
Maximum	1.910	1.802	0.270	3.268
Std.Dev	0.150	0.108	0.057	0.891
Autocorrelation	0.983	0.962	0.917	0.923

B. Correlations				
	P^B_{GV}	P^S_{GV}	P^V_{BS}	TS
P^B_{GV}	1			
P^S_{GV}	0.370	1		
P^V_{BS}	0.097	0.317	1	
TS	0.029	0.266	0.181	1

C. Unit Root Test	
	ADF Statistic
P^B_{GV}	$-4.75**$
P^S_{GV}	-1.65
P^V_{BS}	-3.16^{*}
TS	-3.59^{**}

** Significant at the 1% level
* Significant at the 5% level

Table 3
State Variable Predictive Regressions

Monthly regressions of the realized market excess return, r_M, risk free rate, r_f, and realized Sharpe ratio, SH, on state variable proxies: P^B_{GV}, the (log of the) ratio of the market-to-book ratios for large growth and value firms; P^S_{GV}, the (log of the) ratio of the market-to-book ratios for small growth and value firms; P^V_{BS}, the (log of the) ratio of the market-to-book ratios for value big and small firms; TS, the term spread measured by the difference between the yields of 10 year and 1 year Treasury Bonds (% per year). SH is the market excess return divided by the volatility estimated from an EGARCH model. The sample period is from January 1950 to December 1999. *t*-ratios adjusting for HAC errors are in parentheses.

Dependent Variables	Const.	$P^B_{GV}(-1)$	$P^S_{GV}(-1)$	$P^V_{BS}(-1)$	$TS(-1)$	R^2	S.E.
r_M	5.185	0.507	-3.895	6.664		0.01	4.160
	(1.80)	(0.33)	(1.96)	(1.93)			
r_M	6.780	0.832	-5.481	5.463	0.730	0.03	4.119
	(2.33)	(0.61)	(2.72)	(1.58)	(3.46)		
r_f	-0.356	-.682	1.011	-0.858		0.17	0.263
	(1.30)	(4.78)	(6.09)	(3.83)			
r_f	-0.346	-0.680	1.001	-0.854	0.004	0.16	0.263
	(1.30)	(4.75)	(6.04)	(3.70)	(0.27)		
SH	0.774	0.249	-0.725	1.492		0.00	1.008
	(1.11)	(0.69)	(1.57)	(1.89)			
SH	1.184	0.333	-1.133	1.186	0.188	0.03	0.996
	(1.70)	(1.01)	(2.40)	(1.49)	(3.88)		

Table 4
E-GARCH State Variable Predictive Regressions

Monthly regressions of the realized market excess return, r_M, and realized Sharpe ratio, SH, on state variable proxies: P^B_{GV}, the (log of the) ratio of the market-to-book ratios for large growth and value firms; P^S_{GV}, the (log of the) ratio of the market-to-book ratios for small growth and value firms; P^V_{BS}, the (log of the) ratio of the market-to-book ratios for value big and small firms; TS, the term spread measured by the difference between the yields of 10 year and 1 year Treasury Bonds (% per year). SH is the market excess return divided by the volatility estimated from an EGARCH model. The sample period is from January 1950 to December 1999. *z*-ratios are in parentheses.

	Dependent Variables	Const.	$P^B_{GV}(-1)$	$P^S_{GV}(-1)$	$P^V_{BS}(-1)$	$TS(-1)$	$\ln \sigma^2_{t-1}$	$\frac{\lvert\epsilon_{t-1}\rvert}{\sigma_{t-1}}$	$\frac{\epsilon_{t-1}}{\sigma_{t-1}}$	Adj. R^2
1.	r_M	2.179	1.574	-2.885	4.820					-0.01
		(0.86)	(1.32)	(1.70)	(1.68)					
	$\ln \sigma^2_t$	0.284	-0.153	0.159	-1.011		0.897	0.117	-0.149	
		(1.82)	(2.32)	(1.89)	(4.22)		(27.72)	(1.69)	(4.15)	
2.	r_M	3.672	1.387	-3.716	2.725	0.502				0.01
		(1.46)	(1.17)	(1.96)	(0.84)	(2.54)				
	$\ln \sigma^2_t$	0.161	-0.227	0.503	-1.192	-0.063	0.816	0.100	-0.172	
		(0.79)	(2.37)	(2.95)	(3.60)	(2.44)	(13.80)	(1.26)	(4.00)	
3.	SH	0.848	0.324	-0.843	1.492					-0.01
		(1.32)	(1.22)	(1.81)	(1.81)					
	$\ln \sigma^2_t$	-1.451	0.556	0.425	0.635		-0.470	-0.144	-0.185	
		(1.37)	(1.14)	(0.63)	(0.41)		(2.07)	(1.30)	(3.26)	
4.	SH	1.199	0.433	-1.236	1.100	0.199				0.01
		(1.87)	(1.64)	(2.61)	(1.38)	(4.10)				
	$\ln \sigma^2_t$	-1.312	0.425	0.486	0.609	-0.018	-0.364	-0.221	0.192	
		(1.26)	(0.93)	(0.71)	(0.41)	(0.19)	(1.47)	(1.89)	(3.26)	

Table 5
Return Based and Yield Based Parameter Estimates

This table reports estimates of the parameters of the stochastic process of the investment opportunity set. The return-based estimates (Row 1) are obtained from the following discretized version of equations (5.2) and (5.3):

$$
\begin{aligned}
r_t &= \bar{r}\left(1 - e^{-\kappa_r \Delta t}\right) + e^{-\kappa_r \Delta t} r_{t-\Delta t} + \epsilon_{r,t}, \\
\eta_t &= \bar{\eta}\left(1 - e^{-\kappa_\eta \Delta t}\right) + e^{-\kappa_\eta \Delta t} \eta_{t-\Delta t} + \epsilon_{\eta,t},
\end{aligned}
$$

where r and η are the fitted state variables from the regressions in Table 3.

The yield-based estimates (Row 2) are derived from the Kalman filter applied to the inflation and bond yield data with $\bar{r} = 1.24\%$, $\bar{\pi} = 4.1\%$ and $\bar{\eta} = 0.7$. $\bar{r}$ and $\bar{\pi}$ are the sample means, and $\bar{\eta}$ is slightly above the returned-based estimate of this parameter. For yield-based estimates, the Wald test is for $H_0 : \kappa_\eta = \sigma_\eta = 0$. t-ratios are in parenthesis.

Panel A: Model Parameters

		σ_b	σ_P	κ_r	$\bar{r}$	σ_r	κ_η	$\bar{\eta}$	σ_η	κ_π	σ_π
1	Return Based Estimates			0.48 (3.37)	0.01 (2.41)	0.01	1.16 (5.08)	0.67 (5.55)	0.94		
2	Yield Based Estimates	0.12% (8.55)	3.90% (4.86)	0.14 (19.97)		2.06% (6.77)	0.22 (6.33)		10.5% (6.79)	0.00 (24.63)	0.74% (7.74)

Panel B: Correlations

		$\rho_{r\eta}$	$\rho_{r\pi}$	ρ_{rm}	$\rho_{\eta\pi}$	$\rho_{\eta m}$	$\rho_{\pi m}$	ρ_{Pm}	ML	Wald
1	Return Based Estimates	-0.48								
2	Yield Based Estimates	0.22 (3.29)	-0.10 (13.25)	-0.82 (6.35)	0.42 (11.96)	-0.39 (4.48)	0.22 (6.89)	-0.50 (10.10)	27,119.5	77.5 *

* Significant at 1% level.

Table 6
Relation Between Return-Based and Yield-Based State Variables Estimates

r_{return} and η_{return} are the return-based estimates of the real interest rate and the Sharpe ratio while r_{yield} and η_{yield} are the corresponding yield-based estimates. $t-$ratios are in parentheses.

Dependent Variable	$Constant$	η_{yield}	r_{yield}	Adj. R^2	$S.E.$
η_{return}	0.09	0.73		0.22	0.57
	(0.7)	(5.3)			
r_{return}	0.01		-0.06	0.004	0.01
	(2.8)		(0.9)		

Table 7
FF Portfolio Price Ratios and Returns, and Yield Based State Variable Estimates

Panel A reports regressions of portfolio price ratios on the yield-based state variable estimates. Panel B reports regressions of innovations in r, π, and η on the Fama-French portfolio returns. *t*-ratios, reported in parentheses, are calculated using the Newey-West adjustment for heterogeneity and serial correlation.

A. Equity Portfolio Price Ratios vs. Yield Based State Variable Estimates

	Dependent Variables	Const.	r_{yield}	π_{yield}	η_{yield}	Adj. R^2	S.E.
1.	P_{GV}^{B}	1.54	-1.36	-3.01	-0.07	0.32	0.12
		(26.7)	(2.6)	(8.7)	(2.1)		
2.	P_{GV}^{S}	1.55	-0.98	0.37	0.06	0.09	0.10
		(29.9)	(1.9)	(0.9)	(2.1)		
3.	P_{BS}^{V}	0.19	-0.97	-0.13	-0.02	0.08	0.05
		(7.3)	(3.2)	(0.9)	(1.6)		
4.	P_{BS}^{G}	0.18	-1.35	-3.51	-0.14	0.44	0.12
		(3.8)	(2.6)	(11.4)	(5.7)		

B. Innovations in the Yield Based State Variable Estimates vs. FF Portfolios

	Dependent Variables	$Const.$	$r_m - r_f$	SMB	HML	Adj. R^2	S.E.
1.	$\hat{\epsilon}_r$	0.001	-0.03	-0.002	-0.05	0.04	0.006
		(4.2)	(3.0)	(0.2)	(3.3)		
2.	$\hat{\epsilon}_\pi$	0.000	-0.01	0.01	0.01	0.02	0.003
		(0.6)	(1.5)	(1.6)	(2.1)		
3.	$\hat{\epsilon}_\eta$	-0.000	0.01	0.30	0.29	0.03	0.06
		(0.03)	(0.13)	(2.7)	(2.4)		

Table 8
Summary Statistics on the Valuation Factor

The table reports the variation in the valuation factor, $v \equiv e^{A(\tau)-B(\tau)r-D(\tau)\eta}$, that is attributable to variation in state variables r, $v^r \equiv e^{A(\tau)-B(\tau)r}$, and η, $v^\eta \equiv e^{A(\tau)-D(\tau)\eta}$. Factors are calculated for maturities of 1, 5, and 20 years when the cash flow beta is unity. Under the CAPM assumption, the pricing kernel is $dm/m = -rdt - \eta dz_M$. Under the ICAPM assumption the pricing kernel is $dm/m = -rdt - \eta(\zeta_M dz_M - \zeta_\eta dz_\eta - \zeta_r dz_r)$. For both the CAPM and the ICAPM, the correlations between the innovations in r and η and the market return, ρ_{Mr} and $\rho_{M\eta}$, are the sample estimates, -0.12 and 0.01 respectively. For the ICAPM, the coefficients of the pricing kernel match the yield-based estimates of the parameters $\rho_{mr}(-0.82)$ and $\rho_{m\eta}(-0.39)$ and to satisfy the contraint that $\text{Var}\,(dm/m) = \eta^2$. The sample period is from March 1950 to September 1996. All results are based on the yield-based estimates of state variables and parameters. $\sigma(.)$ is the standard deviation of the time series of values. $\sigma(dv/v)$ is the annualized volatility of the proportional changes in values.

	CAPM			ICAPM		
	v^η	v^r	v	v^η	v^r	v
$\tau = 1$ year	$A(\tau) = -0.011$			$A(\tau) = -0.008$		
	$B(\tau) = 0.933$			$B(\tau) = 0.933$		
	$D(\tau) = 0.131$			$D(\tau) = 0.104$		
Mean	0.883	0.927	0.827	0.906	0.929	0.849
Minimum	0.770	0.886	0.715	0.812	0.889	0.755
Maximum	1.066	0.979	0.989	1.052	0.982	0.980
$\sigma(.)$	0.049	0.015	0.043	0.040	0.015	0.035
$\sigma(dv/v)$	0.027	0.023	0.034	0.022	0.023	0.030
$\tau = 5$ years	$A(\tau) = -0.211$			$A(\tau) = -0.191$		
	$B(\tau) = 3.596$			$B(\tau) = 3.596$		
	$D(\tau) = 0.454$			$D(\tau) = 0.474$		
Mean	0.554	0.631	0.431	0.557	0.644	0.432
Minimum	0.341	0.531	0.257	0.334	0.542	0.252
Maximum	1.048	0.779	0.791	1.082	0.795	0.815
$\sigma(.)$	0.109	0.040	0.079	0.115	0.041	0.084
$\sigma(dv/v)$	0.101	0.096	0.126	0.104	0.095	0.128
$\tau = 20$ years	$A(\tau) = -1.651$			$A(\tau) = -2.079$		
	$B(\tau) = 6.709$			$B(\tau) = 6.709$		
	$D(\tau) = 0.708$			$D(\tau) = 1.115$		
Mean	0.108	0.121	0.067	0.053	0.079	0.033
Minimum	0.050	0.087	0.029	0.015	0.057	0.009
Maximum	1.287	0.179	0.175	0.236	0.116	0.135
$\sigma(.)$	0.034	0.015	0.020	0.028	0.009	0.016
$\sigma(dv/v)$	0.188	0.202	0.226	0.258	0.206	0.278

Figure 1
Time Series of State Variable Proxies

P^B_{GV} is the (log of the) ratio of the market-to-book ratios for large growth and value firms; P^S_{GV} is the (log of the) ratio of the market-to-book ratios for small growth and value firms; P^V_{BS} is the (log of the) ratio of the market-to-book ratios for value big and small firms; TS is the term spread as measured by the difference between the yields on the 10 and 1 year Treasury Bonds (% per year).

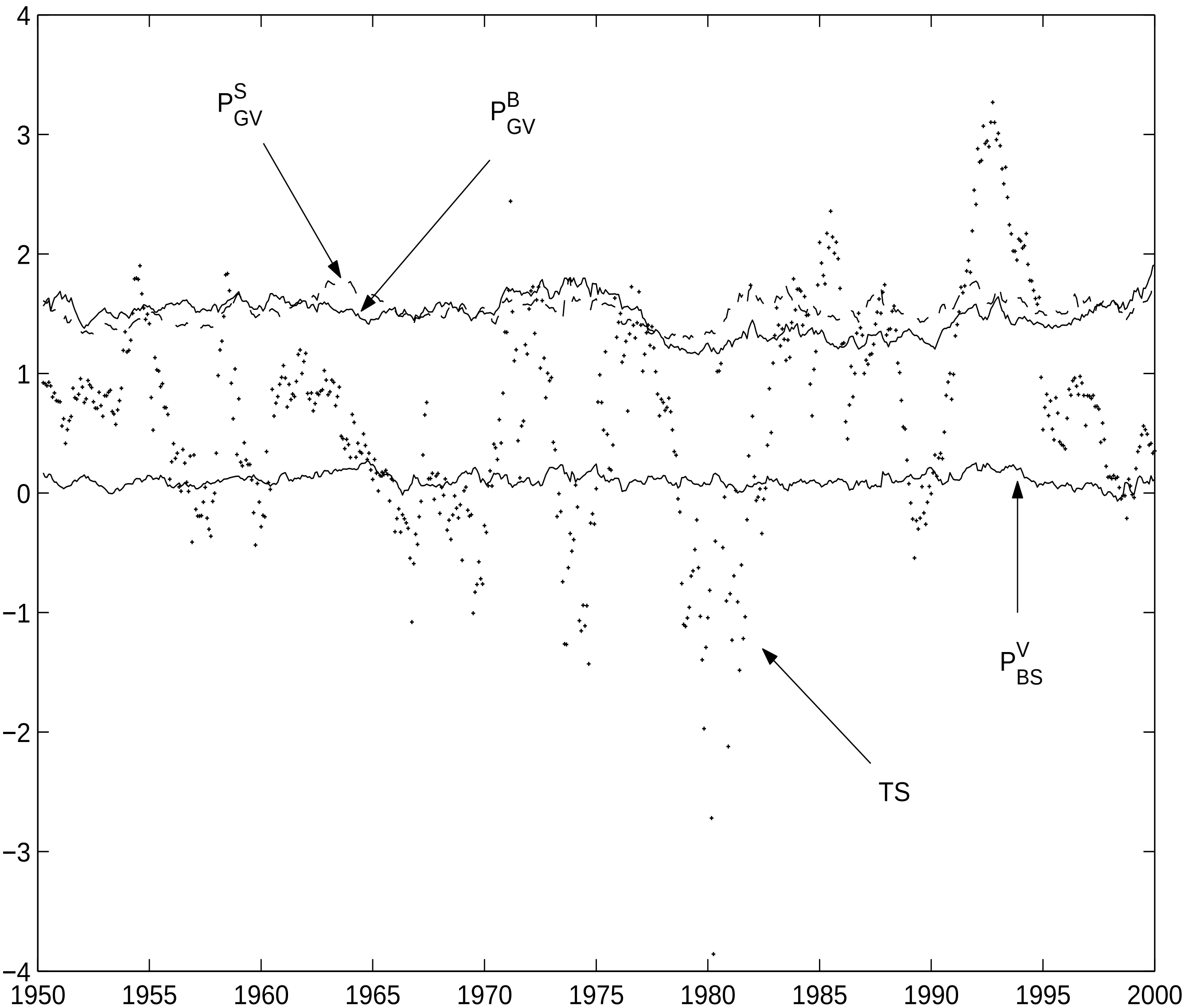


Figure 2
Time Series of Real Interest Rate Estimates

The figure plots two estimated real interest rate series from March 1950 to September 1996: the yield-based estimates are filtered out from the bond yield and inflation data and the return-based estimates are fitted values from predictive regressions of real bill returns. Shaded area indicates periods of U.S. recessions.

Return based estimates - markers; Yield based estimates - solid line.

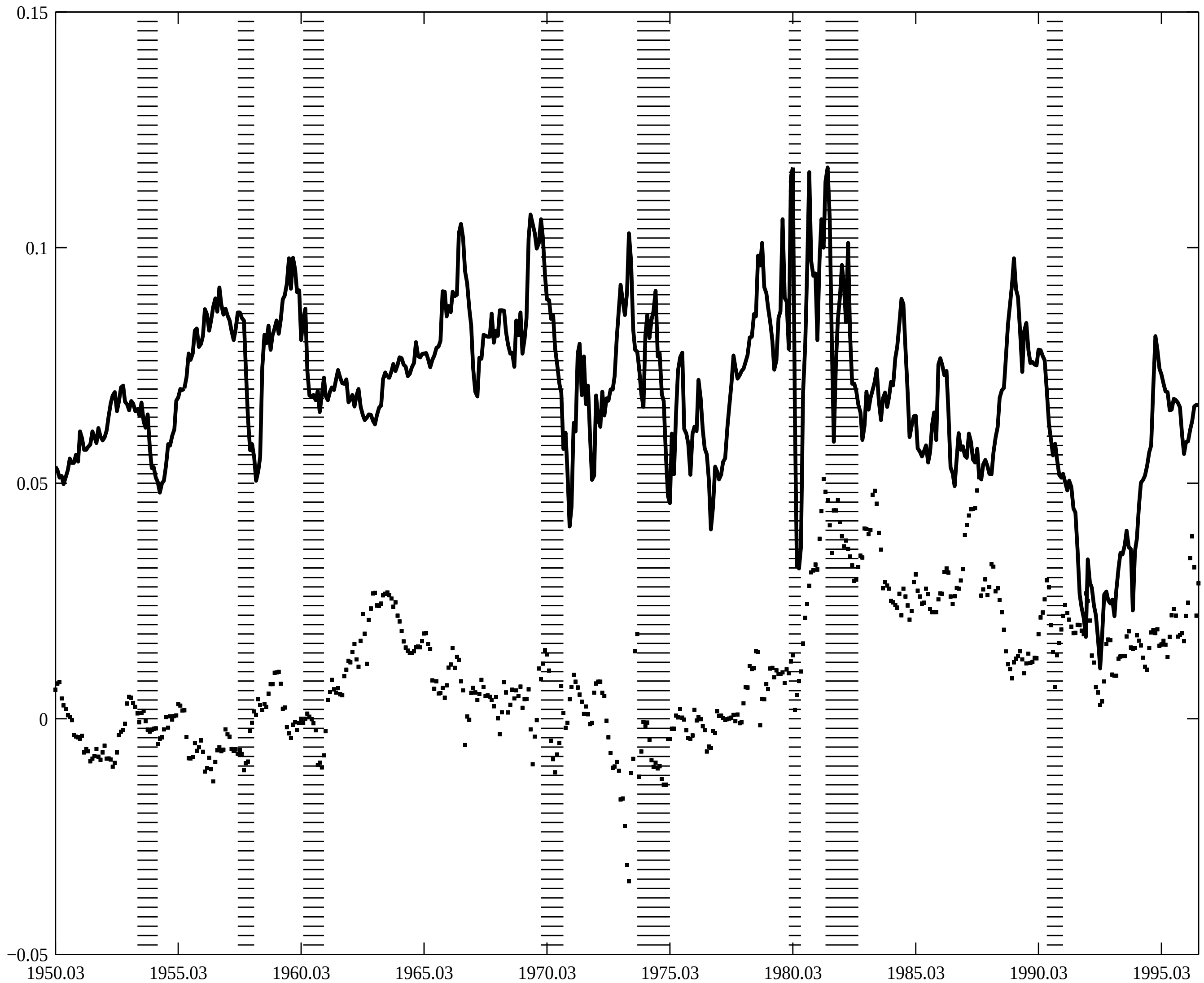


Figure 3
Time Series of Sharpe Ratio Estimates

The figure plots two estimated Sharpe ratio series from March 1950 to September 1996: the yield-based estimates are filtered out from the bond yield and inflation data and the return-based estimates are fitted values from predictive regressions of market excess return. Shaded area indicates periods of U.S. recessions.

Return based estimates - markers; Yield based estimates - solid line.

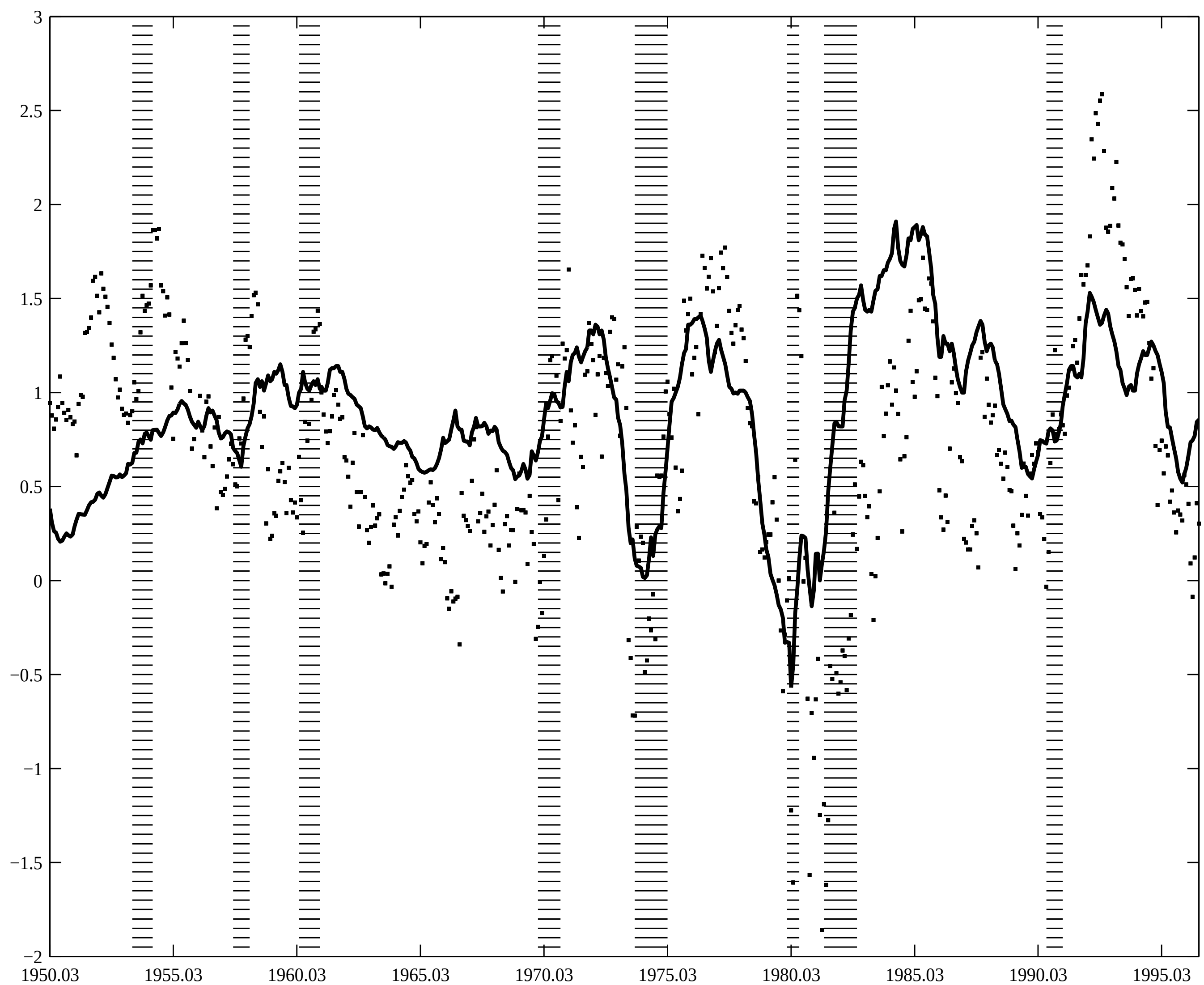
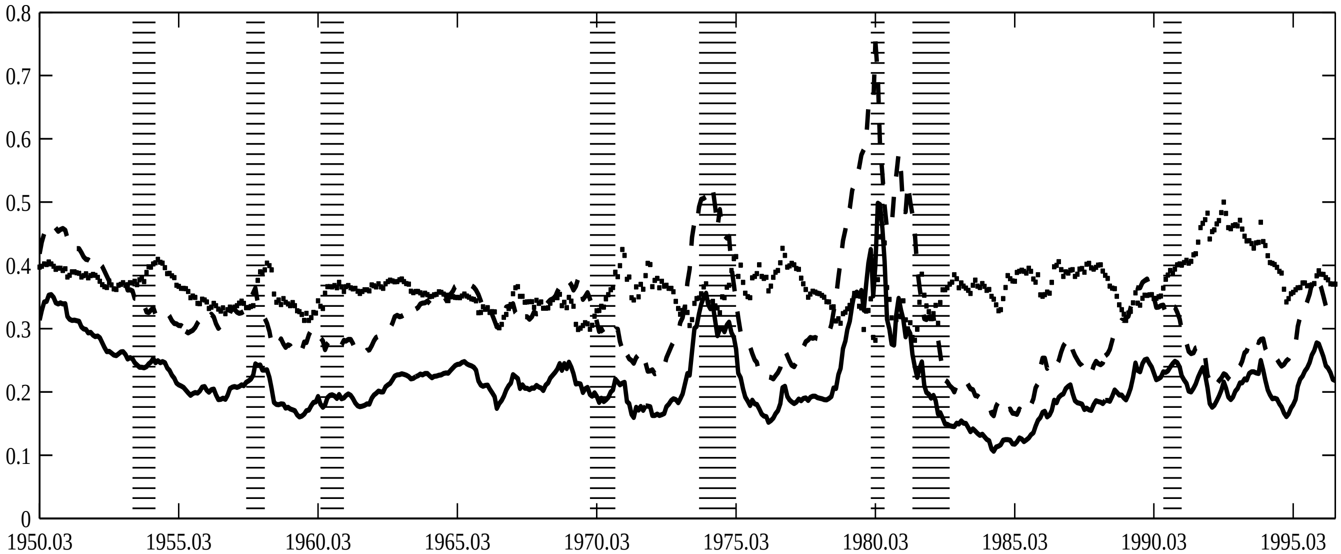


Figure 4
Time Variation in Valuation Factor

The valuation factor is calculated for maturity τ = 10 years and cash flow beta of unity. For both the CAPM and the ICAPM, the correlations between the innovations in r and η and the market return, ρ_{Mr} and $\rho_{M\eta}$, are the sample estimates, -0.12 and 0.01 respectively. For the ICAPM, the coefficients of the pricing kernel match the yield-based estimates of the parameters $\rho_{mr}(-0.82)$ and $\rho_{m\eta}(-0.39)$ and to satisfy the contraint that $\mathrm{Var}\,(dm/m) = \eta^2$. All results are based on the yield-based estimates of state variables and parameters. Shaded area indicates periods of U.S. recessions. $v^\eta = e^{A(\tau)-D(\tau)\eta}$, $v^r = e^{A(\tau)-B(\tau)r}$, and $v = e^{A(\tau)-B(\tau)r-D(\tau)\eta}$.

v^η - dashed line; v^r - markers; v - solid line.

a: Valuation Factor under CAPM



b: Valuation Factor under ICAPM